Nano-optomechanics

Ewold Verhagen

AMOLF
Amsterdam, The Netherlands
www.optomechanics.nl
The push of a photon

Keppler, *De Cometis* (1619)

"Untersuchungen über die Druckkräfte des Lichtes"
The push of a photon

Arthur Ashkin
*Optical tweezers*

Dave Wineland, Rainer Blatt, ...
*Laser cooling and control of ions*
Micro- and nanomechanics

Small mechanical sensors: minute forces or masses affect resonance frequency

Mechanically-assisted signal processing: Exploiting coherence

image: Purdue University/Seyet LLC

Broadcom
Motivations in optomechanics research

• Quantum physics with ‘macroscopic’ mechanical systems

• New ways to control optical and acoustic signals

• Exquisite sensors and mechanical quantum transducers
Outline

• Optical forces

• Optomechanical measurement

• Quantum description of optomechanical interaction

• State transfer & laser cooling
Optical forces

Formally: force per unit volume $\mathbf{f}$ given by Maxwell stress tensor $\sigma$

$$
\mathbf{f} + \epsilon_0 \mu_0 \frac{\partial \mathbf{S}}{\partial t} = \nabla \cdot \sigma \quad \quad \sigma_{ij} = \epsilon_0 E_i E_j + \frac{1}{\mu_0} B_i B_j - \frac{1}{2} \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) \delta_{ij}
$$
Optical forces

**Gradient & scattering force**

\[ \langle \mathbf{F} \rangle = \frac{\text{Re} \alpha}{4} \nabla |\mathbf{E}_0|^2 + \frac{\text{Im} \alpha}{2} \mathbf{k} |\mathbf{E}_0|^2 \]

Gaussian beam
A cavity optomechanical system
Moving boundaries vs photoelastic effect

**Silicon nitride membranes**


**Trapped particles**

Kiesel et al, PNAS 110, 14180 (2013)

**Brillouin optomechanics**


**Raman (molecular) optomechanics**

Deformable on-chip photonic cavities

Ring resonators

Verhagen et al., Nature 482, 63 (2012)

Photonic crystal nanobeams

The optomechanics zoo

A cavity optomechanical system

\[
\omega_c (x) = \omega_c - G x(t)
\]

\[
G = - \frac{\partial \omega}{\partial x} = \frac{\omega_c}{L}
\]

force: \( F_{rp} = \hbar G n_c \)

phase shift:
\[
\delta \varphi \approx 4 \frac{G}{\kappa} \delta x
\]

cavity linewidth

optical phase

\( G x(t) \)

\( \varphi(t) \)

\( \omega_{\text{laser}} \)

optical frequency
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Macro-optomechanics

Abbott et al., PRL 116, 061102 (2016)
Measuring displacement

Optomechanical phase shift: \( \delta \phi \approx 4 \frac{\partial \omega / \partial x}{\kappa} \delta x \)

Mechanical transduction:

Cavity linewidth
Measuring displacement

Optomechanical phase shift:

\[ \delta \phi \approx 4 \frac{\partial \omega}{\partial x} \frac{\delta x}{\kappa} \]

Mechanical oscillator undergoes Brownian motion

\[ \tilde{x}(\omega) = \frac{1}{\sqrt{\tau}} \int_0^\tau x(t) e^{i \omega t} \, dt \]

average

\[ S_{xx}(\omega) = \lim_{\tau \to \infty} \langle \left| \tilde{x}(\omega) \right|^2 \rangle \]

noise spectral density
Measuring displacement

Measured phase fluctuation spectrum:

Measured displacement fluctuation spectrum:

\[
\Gamma_m
\]

\[
\frac{1}{2} m \Omega_m^2 \langle x^2 \rangle = \frac{k_b T}{2}
\]

\[
\tilde{x}(\omega) = \frac{1}{\sqrt{\tau}} \int_0^\tau x(t)e^{i\omega t} dt
\]

average

\[
S_{xx}(\omega) = \lim_{\tau \to \infty} \langle |\tilde{x}(\omega)|^2 \rangle
\]

noise spectral density

measurement imprecision (photon shot noise)
The Standard Quantum Limit

SQL: Manifestation of Heisenberg’s uncertainty principle for continuous measurement

\[ x_{zpf} = \sqrt{\frac{\hbar}{2m\Omega_m}} \approx 10^{-14} - 10^{-18} \text{ m} \]

Spectral density of zero-point fluctuations

![Graph showing the relationship between spectral density and frequency, illustrating quantum backaction and shot noise imprecision.](image)
The Standard Quantum Limit

SQL: Manifestation of Heisenberg’s uncertainty principle for continuous measurement

\[ \bar{n}_{\text{th}} = \frac{k_B T}{\hbar \Omega_m} \]

![Graph showing the Standard Quantum Limit](image)

- **thermal+quantum motion**
- **quantum backaction**
- **shot noise imprecision**
Observing quantum backaction

- Strong ‘signal’ beam is source of RP shot noise
- Weak ‘meter’ beam used to read out motion
Cooling with measurement-based feedback

- Feeding back on slope of measured displacement
- Radiation pressure of 2nd laser to provide feedback (damping)

\[ \bar{n}_{th} = \frac{k_B T}{\hbar \Omega_m} \]

Cooling with measurement-based feedback

\[ \frac{S_{xx}}{S_{xx}^{\text{SP}}}, \quad \frac{P}{P_{\text{SQL}}} \]

- thermal + quantum motion
- quantum backaction
- shot noise imprecision
Sideband picture

Mechanical transduction:

\[ x(t) \]

\[ \varphi(t) \]
Sideband picture

Mechanical transduction:

Generating optical laser sidebands

At low thermal phonon occupancy, the upper and lower mechanical sidebands are asymmetric
Observing mechanical zero-point motion

At low thermal phonon occupancy, the upper and lower mechanical sidebands are asymmetric.

*Safavi-Naeini et al., PRL 108, (2012) 033602*
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• Optical forces
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  • State transfer & laser cooling
The optomechanical Hamiltonian

Optical cavity frequency shift:

$$\omega_c(x) = \omega_c - Gx$$

$$\hat{H} = \hbar \omega_c (\hat{x}) \hat{a}^\dagger \hat{a} + \hbar \Omega_m \hat{b}^\dagger \hat{b}$$

$$\hat{H} = \hbar \omega_c \hat{a}^\dagger \hat{a} + \hbar \Omega_m \hat{b}^\dagger \hat{b} - \hbar G \hat{a}^\dagger \hat{a} \hat{x}$$

$$\hat{E} = E_{\text{vac}} (\hat{a} + \hat{a}^\dagger)$$

$$\hat{x} = x_{\text{zpf}} (\hat{b} + \hat{b}^\dagger)$$

$$\hat{n}_c = \hat{a}^\dagger \hat{a}$$

radiation pressure force
The optomechanical Hamiltonian

Optical cavity frequency shift:

$$\omega_c(x) = \omega_c - Gx$$

vacuum optomechanical coupling rate:

$$g_0 \equiv \frac{\partial \omega}{\partial x} x_{zp}$$
Linearized optomechanics

\[ \hat{H}_{\text{int}} = \hbar g_0 \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger) \]

We can enhance the coupling using a strong coherent laser field at frequency \( \omega_l = \omega_c + \Delta \)

Write optical field as (constant) coherent field \( \alpha \) plus fluctuations:

\[ \hat{a}(t) \rightarrow \alpha + \hat{a}(t) \quad \text{with} \quad |\alpha| = \sqrt{n_c} \quad \text{and} \quad \langle \hat{a}(t) \rangle = 0 \]

\[ \hat{a}^\dagger \hat{a} \rightarrow |\alpha|^2 + \alpha \hat{a}^\dagger + \alpha^* \hat{a} + \hat{a}^\dagger \hat{a} \quad \text{linearization} \]

\[ \hat{H}_{\text{int}} = \hbar g_0 (\alpha^* \hat{a} + \alpha \hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger) \]

\[ \hat{H}_{\text{int}} = \hbar g (\hat{a} + \hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger) \quad \text{with} \quad g \equiv g_0 \alpha \in \mathbb{R} \]
Dynamical backaction

\[ \hat{H}_{\text{int}} = \hbar g (\hat{a} + \hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger) \]

with \( g \equiv g_0 \alpha \in \mathbb{R} \)
Dynamical backaction

**Blue-detuned laser drive:**

\[ \hat{H}_{\text{int}} = \hbar g (\hat{a}^{\dagger} \hat{b}^{\dagger} + \hat{a} \hat{b}) \]

‘two-mode squeezing’ Hamiltonian

- Amplification of motion
- Mechanical self-oscillations
- Light-motion **entanglement**

**Red-detuned laser drive:**

\[ \hat{H}_{\text{int}} = \hbar g (\hat{a}^{\dagger} \hat{b} + \hat{a} \hat{b}^{\dagger}) \]

‘beamsplitter’ Hamiltonian

- Damping of resonator
- Possible ground state cooling
- Light-motion **state transfer**

\[ \hat{H}_{\text{int}} = \hbar g (\hat{a} + \hat{a}^{\dagger})(\hat{b} + \hat{b}^{\dagger}) \quad \text{with} \quad g \equiv g_0 \alpha \in \mathbb{R} \]
Outline

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Laser cooling of motion

\[ \hat{H}_{\text{int}} = \hbar g (\hat{a} + \hat{a}^{\dagger})(\hat{b} + \hat{b}^{\dagger}) \]

with \( g \equiv g_0 \alpha \in \mathbb{R} \)

Red-detuned laser drive:

- Damping of resonator
- Possible ground state cooling
- Light-motion state transfer

\[ \Gamma_{\text{opt}} = \frac{4g^2}{\kappa} \]

cooling rate:

- Optical bath \( n_\text{th} \approx 0 \)
- Mechanical bath \( n_\text{th} \approx k_B T/\hbar \Omega_m \)
Laser cooling of motion

**Cooling rate:**

\[ \Gamma_{\text{opt}} = \frac{4g^2}{\kappa} \]

**Mechanical bath**

\[ n_{\text{th}} \approx \frac{k_B T}{\hbar \Omega_m} \]

**Optical bath**

\[ n_{\text{th}} \approx 0 \]

**Red-detuned laser drive:**

**Cooperativity**

\[ C = \frac{4g^2}{\kappa \Gamma_m} \]

**Interaction Hamiltonian**

\[ \hat{H}_{\text{int}} = \hbar g (\hat{a} + \hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger) \]

with \( g \equiv g_0 \alpha \in \mathbb{R} \)
Laser cooling of motion

\[ \Gamma_{\text{eff}} = \Gamma_m + \Gamma_{\text{opt}} \]

Blue detuning, \( C > 1 \):
\[ \rightarrow \text{parametric oscillations} \]

\[ \hat{H}_{\text{int}} = \hbar g (\hat{a}^{\dagger} \hat{b}^{\dagger} + \hat{a} \hat{b}) \]
Laser cooling of motion

Red-detuned laser drive:

\[ C = \frac{4g^2}{\kappa \Gamma_m} \]

Cooperativity

Ground state cooling \((n_{\text{eff}} < 1)\) when:

\[ \Gamma_{\text{opt}} > \gamma \]

Quantum cooperativity:

\[ C_q \equiv \frac{4g^2}{\kappa \gamma} > 1 \]
Cooling in electromechanical systems

Capacitively coupled nanodrum coupled to a superconducting resonator

$T = 15 \text{ mK}$

Teufel et al., Nature 471, 204 (2011)
Cooling in electromechanical systems

Capacitively coupled nanodrum coupled to a superconducting resonator

Teufel et al., Nature 471, 204 (2011)
Optomechanically induced transparency

Red-detuned laser drive, near-resonant probe laser:

Cooperativity

\[ C = \frac{4g^2}{\kappa \Gamma_m} \]

Optomechanically induced transparency

bandwidth:

\[ C \Gamma_m \leq \kappa \]

probe frequency
More than two modes: wavelength conversion

Two modes & two red-detuned drives: photon-phonon-photon transfer

Less than 1 added noise quantum for $C_{q(1,2)} > 1$
Microwave-to-optical conversion

Microwave-to-optical conversion

Resonant excitation of surface acoustic waves, conversion to photons

Balram et al., Nat. Photon. 10, 346 (2016)
Coupling to \textit{nonlinear} quantum systems


Satzinger et al., Nature 563, 661 (2018)

Single-phonon control of surface acoustic wave resonator with transmon qubit
Conclusions

- Optomechanics: intersecting photonics, nanotechnology, quantum optics

- Addressing fundamental questions: quantum measurement and macroscopic quantum mechanics

- Simple interactions yield wide range of effects: application potential in various regimes; $(\Omega_m \sim \text{Hz} – \text{THz}; \quad \omega_c \sim \text{GHz} – \text{THz})$