

Introduction to Topological Photonics

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AMOLF Nanophotonics Summer School, June 2019



The group



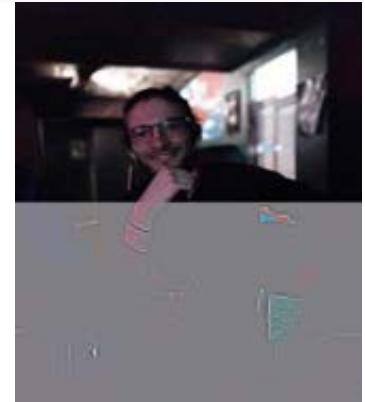
Jiho Noh



Jonathan Guglielmon



Sachin Vaidya



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Dr. Wladimir Benalcazar



Dr. Sebabrata Mukherjee



Kanchita
Klangboonkrong



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Part 1: Introduction to Photonic Topological Insulators

[collaboration with group of M. Segev and A. Szameit]

Nobel prize for physics: 2016



Photo: A.
Mahmoud

**David J.
Thouless**

Prize share: 1/2



Photo: A.
Mahmoud

**F. Duncan
M. Haldane**

Prize share: 1/4



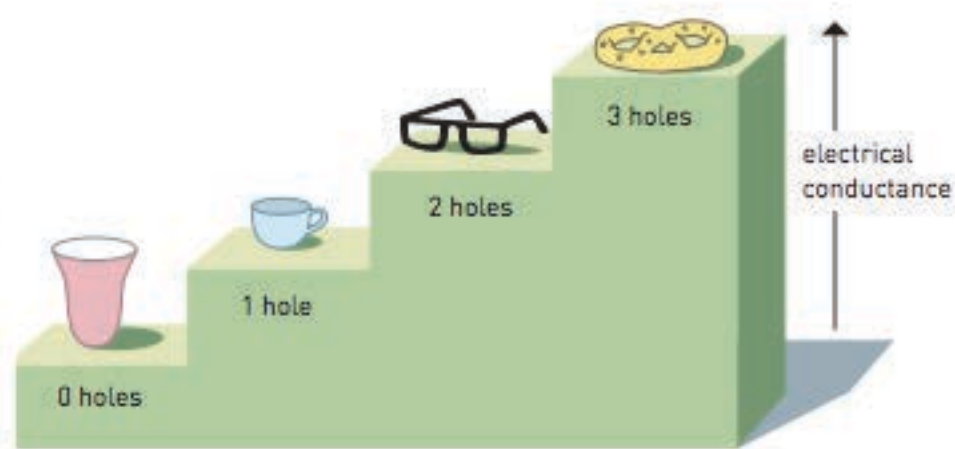
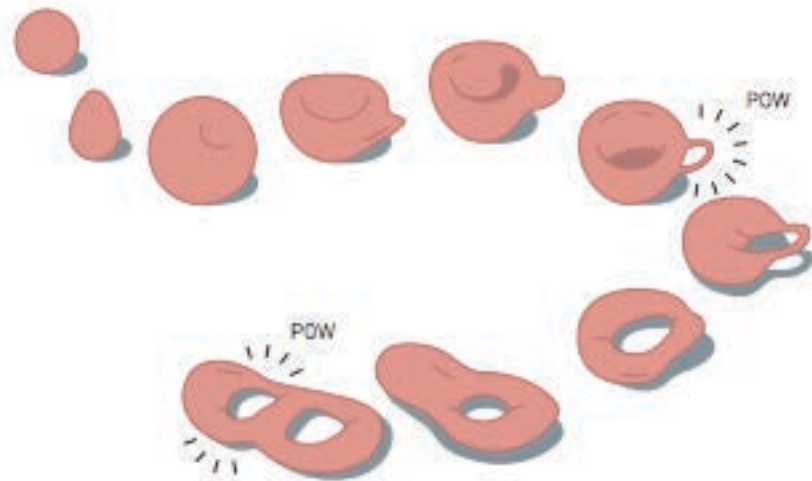
Photo: A.
Mahmoud

**J. Michael
Kosterlitz**

Prize share: 1/4

“For theoretical discoveries of topological phase transitions and topological phases of matter.”

Topological physics



Royal Swedish Academy

A system described by a topological number must be robust

Quantum Hall effect

VOLUME 45, NUMBER 6

PHYSICAL REVIEW LETTERS

11 AUGUST 1980

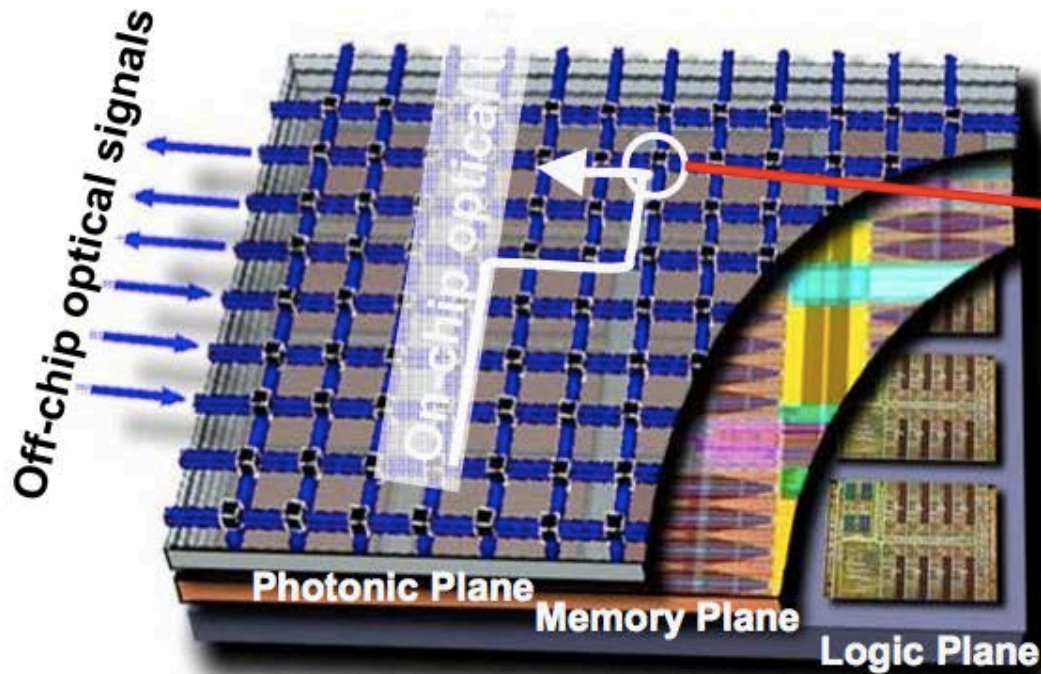
New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

K. v. Klitzing

*Physikalisches Institut der Universität Würzburg, D-8700 Würzburg, Federal Republic of Germany, and
Hochfeld-Magnetlabor des Max-Planck-Instituts für Festkörperforschung, F-38042 Grenoble, France*



Motivation: bring topological robustness into photonics



Optical Switch Network

off

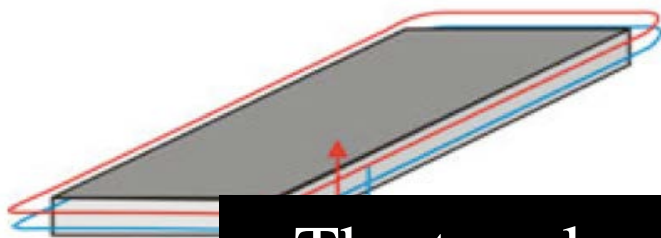
on



Photonic layer not only connects various cores, but also routes the traffic

What are topological insulators?

- Topological insulators are insulators in the bulk but metallic on the edges.
- Most importantly: the edge states are scatter-free!



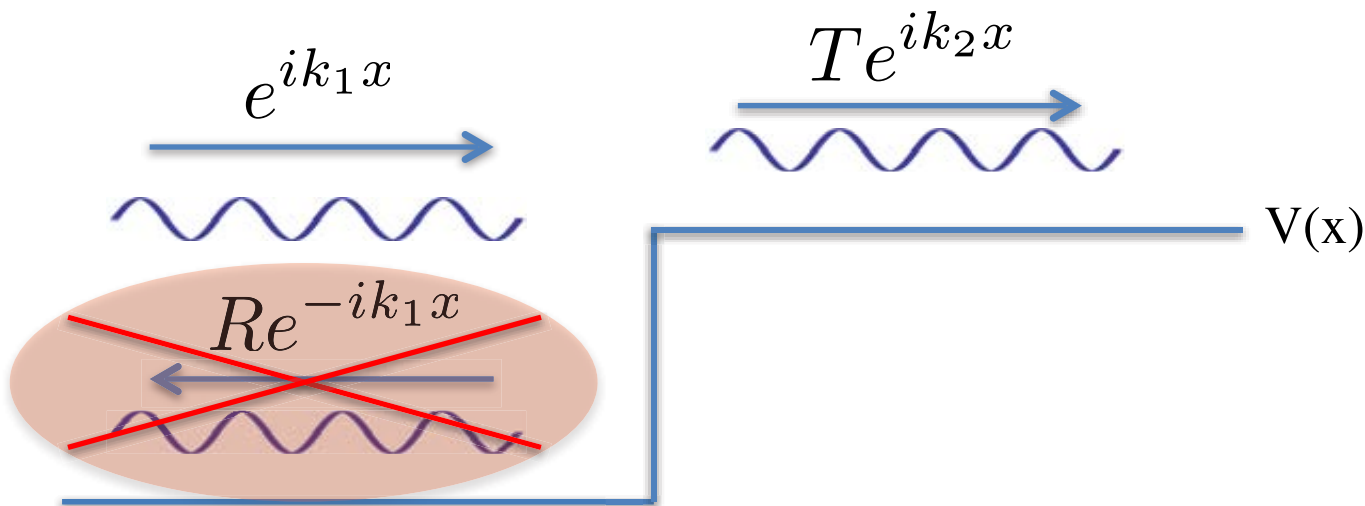
Kane and Mele, Phys. Rev. Lett. 95, 226801 (2005)

Hughes et al., Science 314, 5806, 1757-1761 (2006)

Hsieh et al., Nature 452, 970-974 (2008)

The topological robustness arises from the edge states

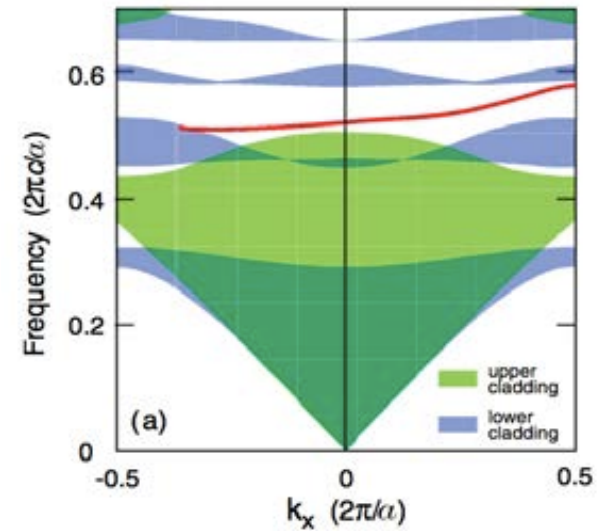
- Why?



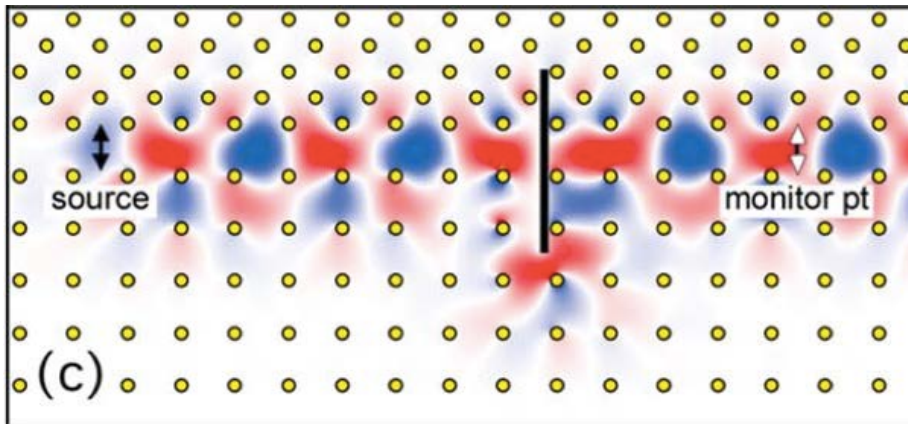
M. Soljacic's microwave experiment: 2009

Raghu and Haldane, 2008

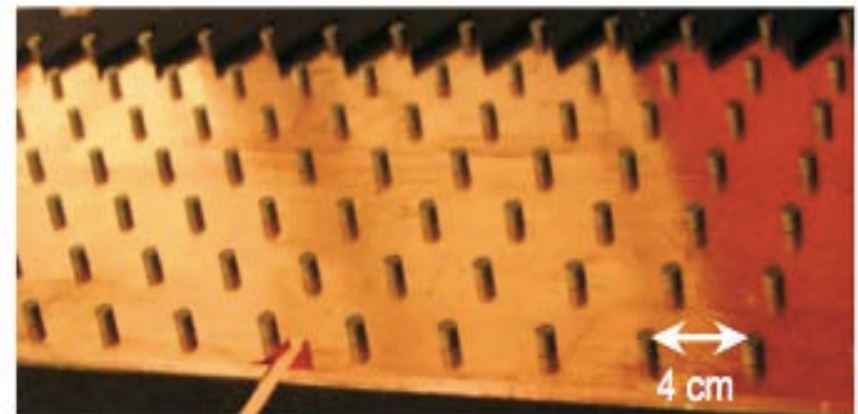
$$\tilde{\mu} = \begin{pmatrix} \mu_1 & i\mu_2 & 0 \\ -i\mu_2 & \mu_1 & 0 \\ 0 & 0 & \mu_1 \end{pmatrix}$$



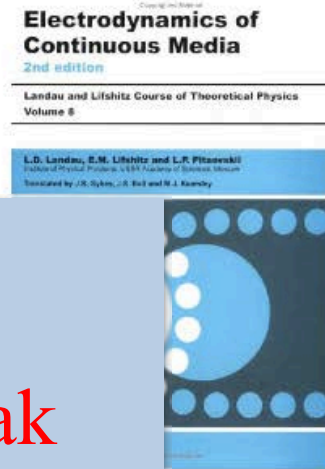
Wang et. al., PRL (2008)



Wang et. al., Nature (2009)



Magnetism in photonics



The big question:
How can we strongly break
time-reversal symmetry
in optics?

Thus there is
onward, and in
and \mathbf{H} in this frequency range would be an effect. Furthermore, the same is true for

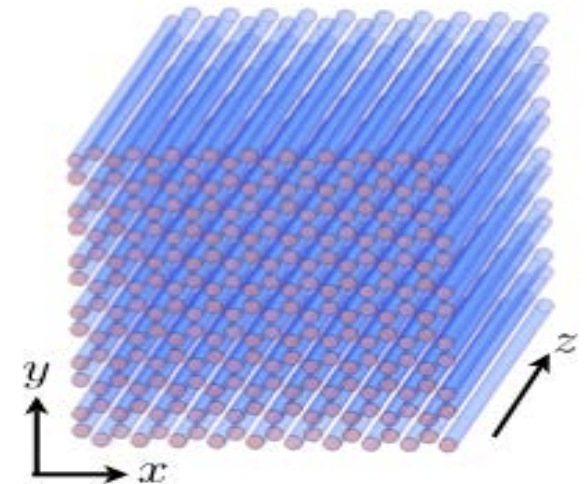
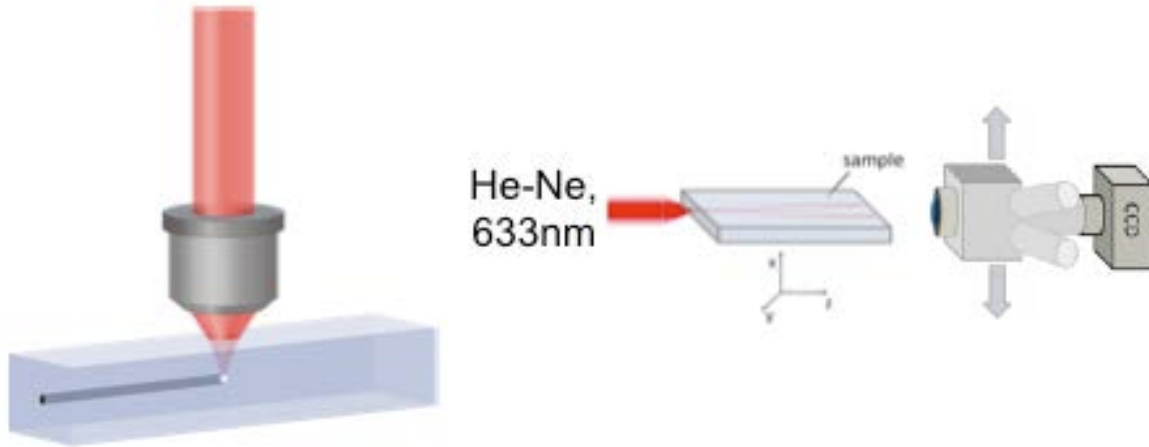
ical frequencies
ish between \mathbf{B}
ame is true for

Magnetic response is inherently weak at optical frequencies

- (1) Hafezi, Demler, Lukin, Taylor, Nature Phys. (2011): CROWS [+ Nature Photon. (2013)]
- (2) Umucalilar and Carusotto, PRA (2011): using spin as polarization in PCs
- (3) Fang, Yu, Fan, Nature Photon. (2012): electrical modulation of refractive index in PCs
- (4) Khanikev et. al. Nature Mat. (2012): birefringent metamaterials [+ Nature Mater. (2016)]

Experimental system: photonic lattices

Array of coupled waveguides



Peleg et. al., PRL (2007)

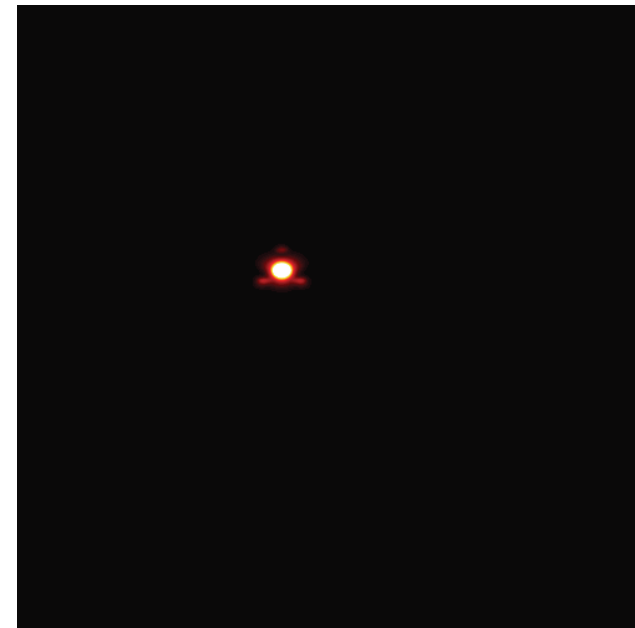
Envelope approximation for electric field:

$$\mathbf{E}(x, y, z) = \hat{x}\psi(x, y, z)e^{i(k_0z - \omega t)}$$

$$|\partial_z^2 \psi| \ll 2k|\partial_z \psi|$$

Paraxial Schrödinger equation:

$$i\partial_z \psi = -\frac{1}{2k} \nabla^2 \psi - \frac{k}{n_0} \Delta n(x, y, \mathbf{z}) \psi$$



Derivation of paraxial approximation

$$\text{Maxwell} \quad \nabla \times \nabla \times \mathbf{E} = \varepsilon(x, y, z) \left(\frac{\omega}{c} \right)^2 \mathbf{E}$$

$$\text{id.} \quad \nabla \times \nabla \times \mathbf{E} = -\nabla^2 \mathbf{E} + \nabla (\nabla \cdot \mathbf{E})$$

$$\text{Maxwell} \quad \nabla \cdot (\varepsilon \mathbf{E}) = \nabla \varepsilon \cdot \mathbf{E} + \varepsilon \nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{E} = -\frac{\nabla \varepsilon \cdot \mathbf{E}}{\varepsilon} = -\nabla \ln \varepsilon \cdot \mathbf{E}$$

$$-\nabla^2 \mathbf{E} - \nabla (\nabla \ln \varepsilon \cdot \mathbf{E}) = \varepsilon(x, y, z) \left(\frac{\omega}{c} \right)^2 \mathbf{E}$$

Derivation of paraxial approximation

$$-\nabla^2 \mathbf{E} = \varepsilon(x, y, z) \left(\frac{\omega}{c}\right)^2 \mathbf{E}$$

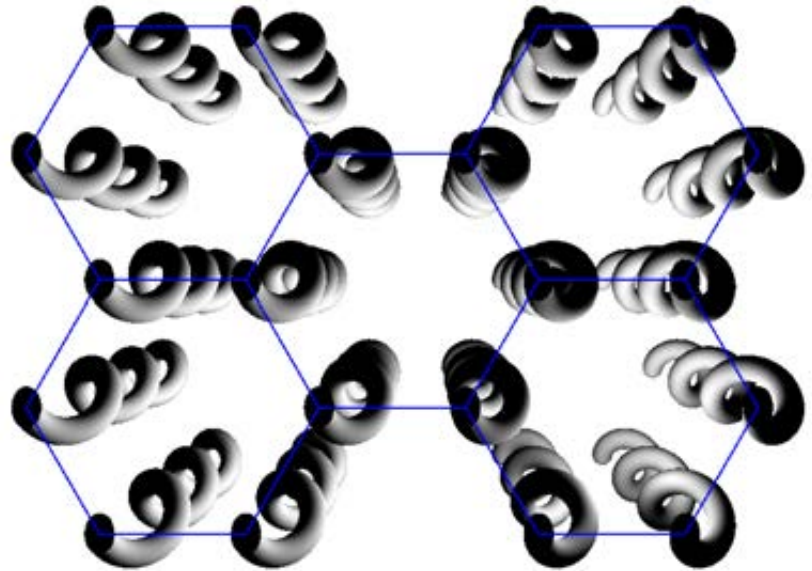
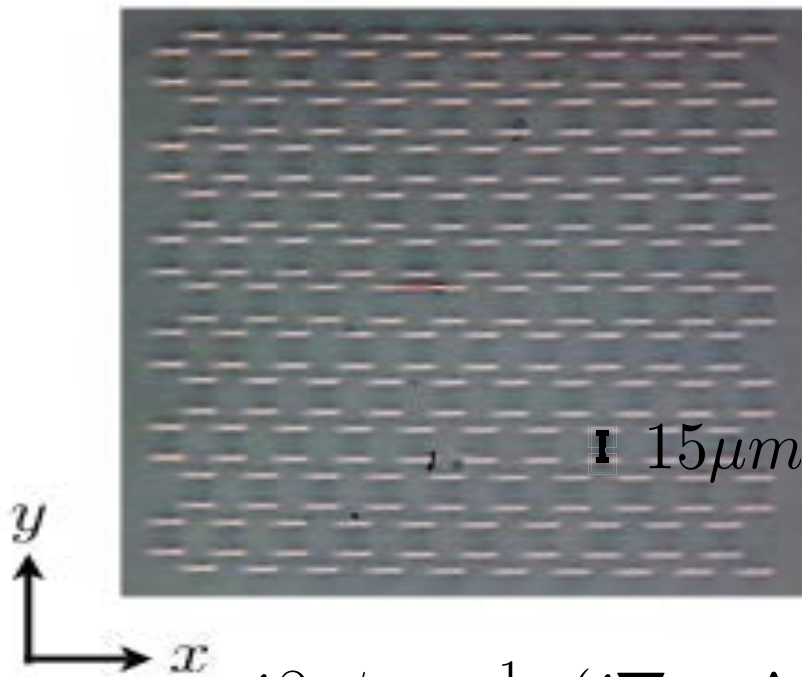
envelope approx: $\mathbf{E} = \psi(x, y, z) e^{ik_0 z} \hat{x}$ (with $k_0 = \sqrt{\varepsilon_1} \omega / c$)

$$\underbrace{-(\partial_x^2 + \partial_y^2)\psi}_{\nabla_{\perp}^2 \psi} - \underbrace{\partial_z^2 \psi}_{\rightarrow 0} - 2ik_0 \partial_z \psi + \underbrace{k_0^2 \psi}_{0} = \varepsilon_1 \left(\frac{\omega}{c}\right)^2 \psi + \Delta\varepsilon(x, y, z) \left(\frac{\omega}{c}\right)^2$$

$$i\partial_z \psi = -\frac{1}{2k_0} \nabla_{\perp}^2 \psi - \frac{k_0}{2} \Delta\varepsilon(x, y, z)$$

$$i\partial_z \psi = -\frac{1}{2k_0} \nabla_{\perp}^2 \psi - \frac{k_0}{n_0} \Delta n(x, y, z)$$

Helical rotation induces a gauge field



$$i\partial_z\psi = \frac{1}{2k_0} (i\nabla + \mathbf{A}(z))^2 \psi - \frac{k_0\Delta n(x,y)}{n_0}\psi - \frac{k_0R^2\Omega^2}{2}\psi$$

$$A(z) = k_0R\Omega(\sin \Omega z, \cos \Omega z)$$

$$x' = x + R \cos \Omega z$$

$$y' = y + R \sin \Omega z$$

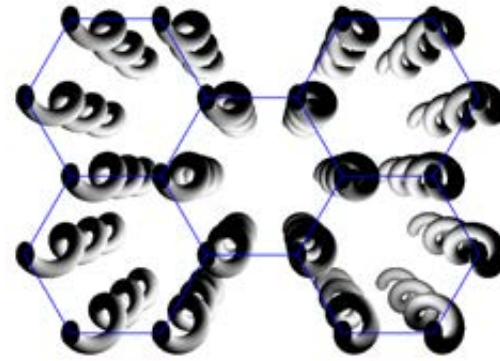
$$z' = z$$

$$\mathcal{H}(z) = \sum_{m, \langle n \rangle} e^{i\mathbf{A}(z) \cdot \mathbf{r}_{mn}} \psi_n^\dagger \psi_m$$

- Floquet TIs: Kitagawa et al., PRB (2010); Lindner et al., Nature Phys. (2011).

Gauge field through helicity

$$\begin{aligned}x' &= x + R \cos \Omega z \\y' &= y + R \sin \Omega z \\z' &= z\end{aligned}$$



$$\partial_z = \partial'_z - R\Omega \sin(\Omega z) \partial'_x + R\Omega \cos(\Omega z) \partial'_y$$

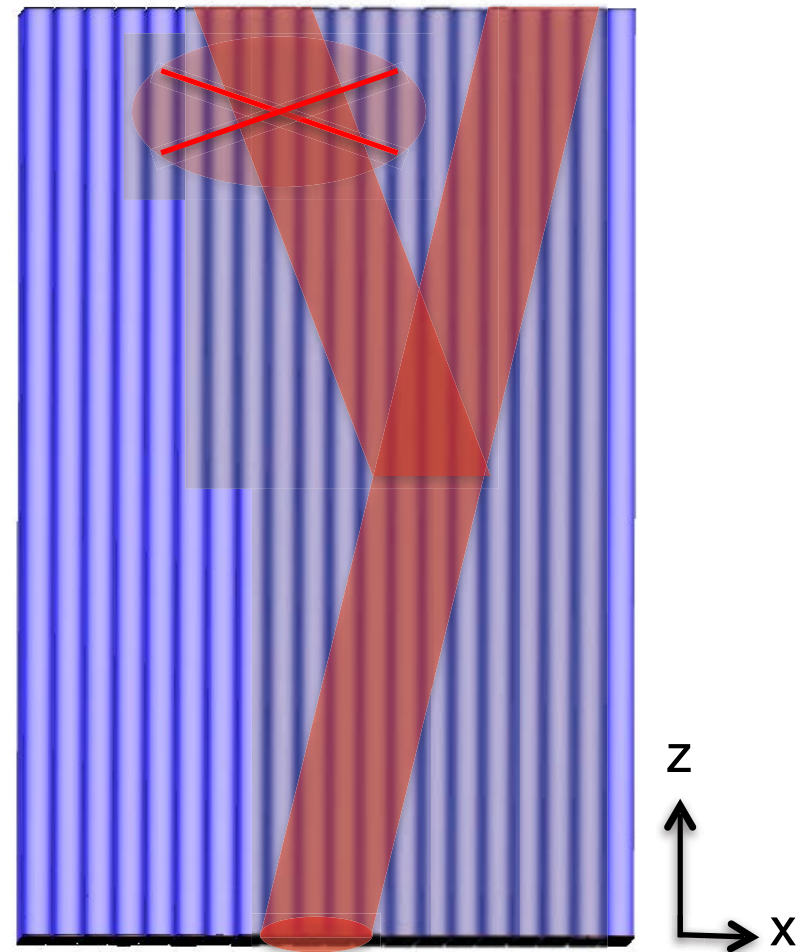
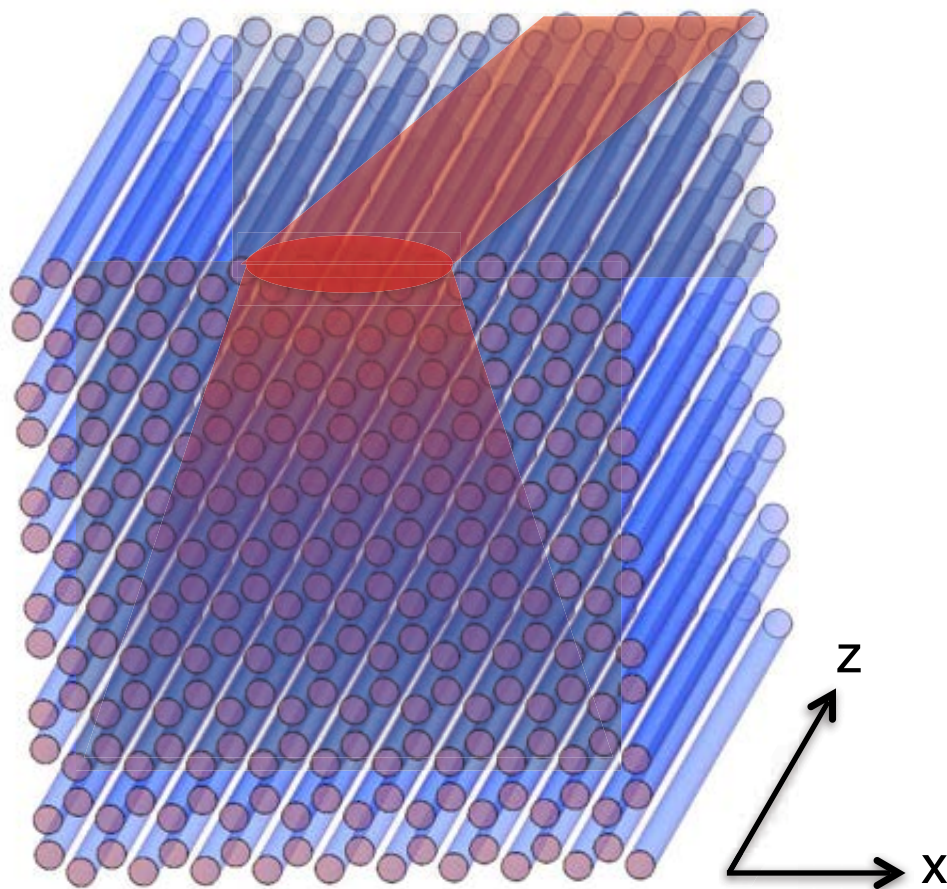
Complete the square

$$i\partial'_z \psi = \frac{1}{2k_0} (i\partial'_\perp - k_0 R\Omega (-\sin \Omega z, \cos \Omega z))^2 \psi - \frac{k_0 R^2 \Omega^2}{2} \psi$$

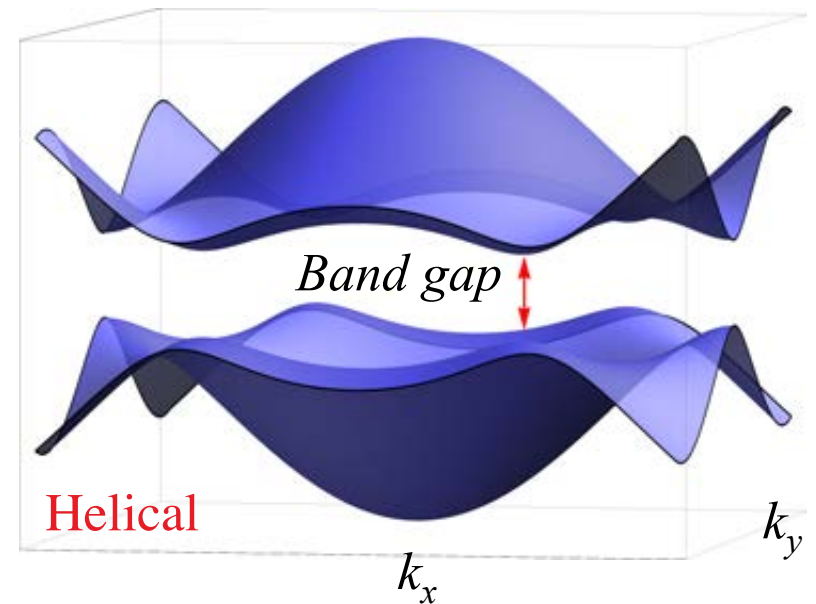
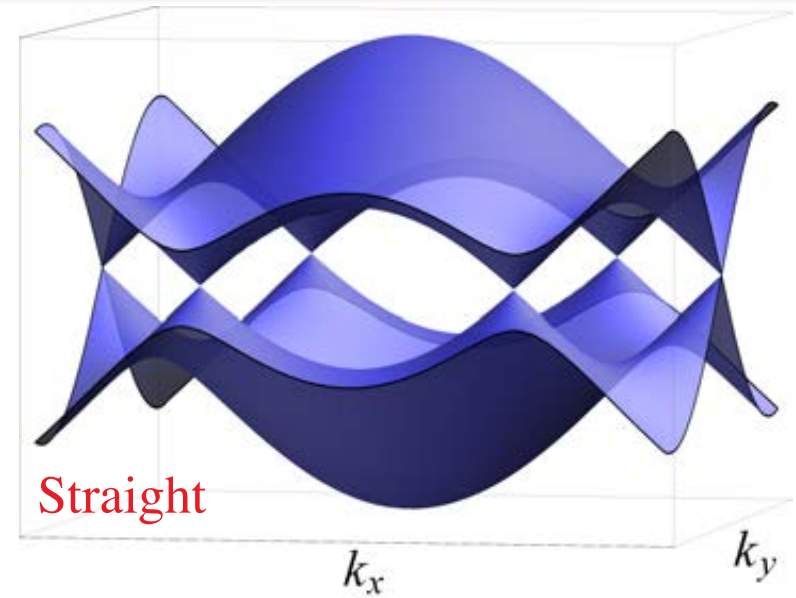
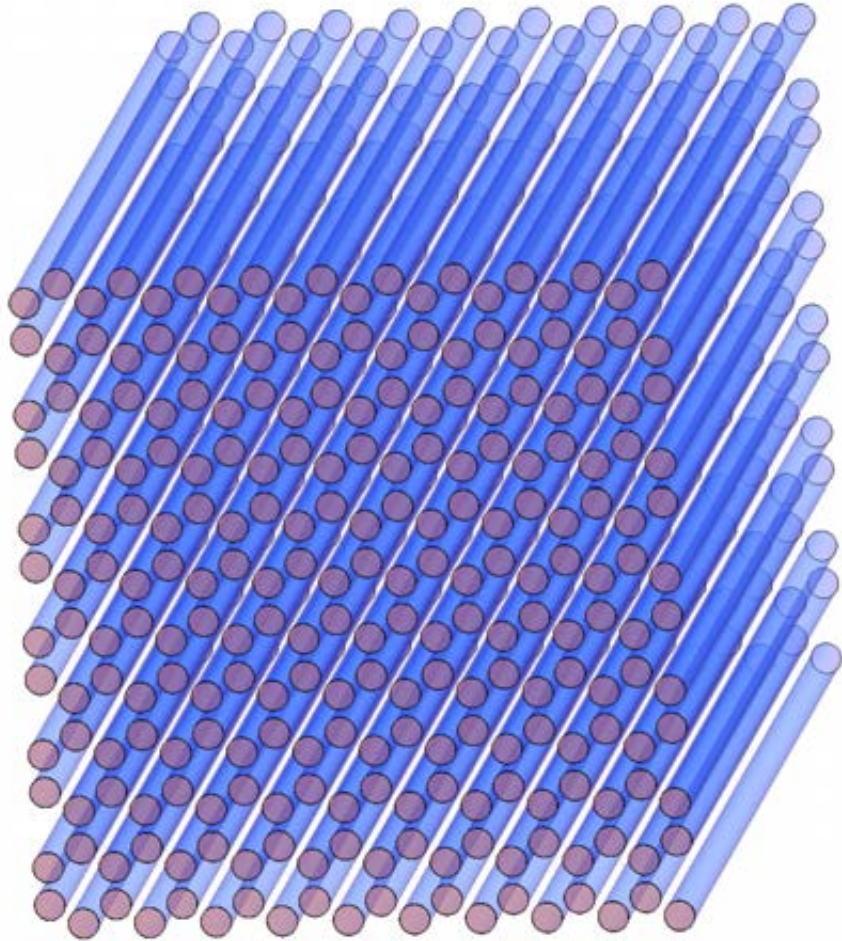
$$\mathbf{A} = k_0 R\Omega (-\sin \Omega z, \cos \Omega z)$$

Experimental system: photonic lattices

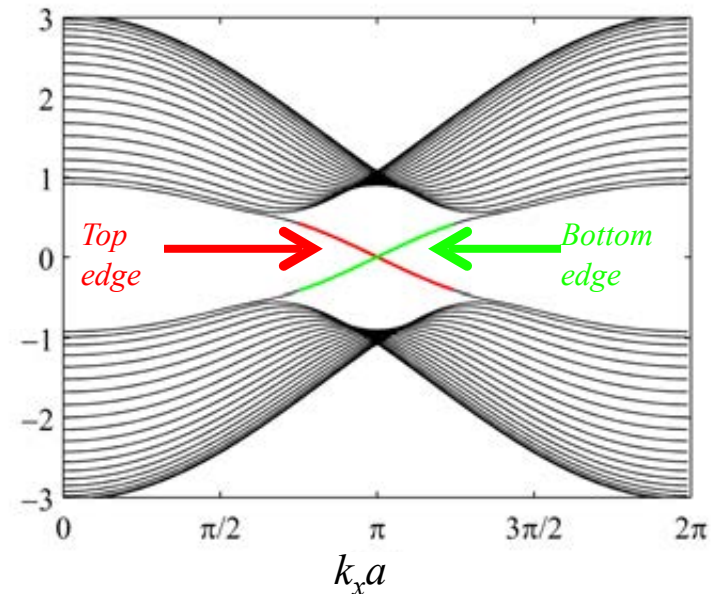
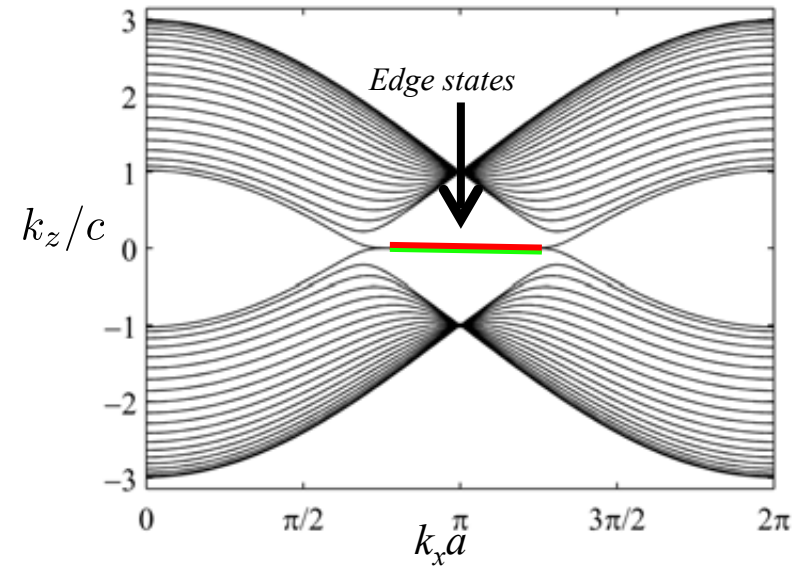
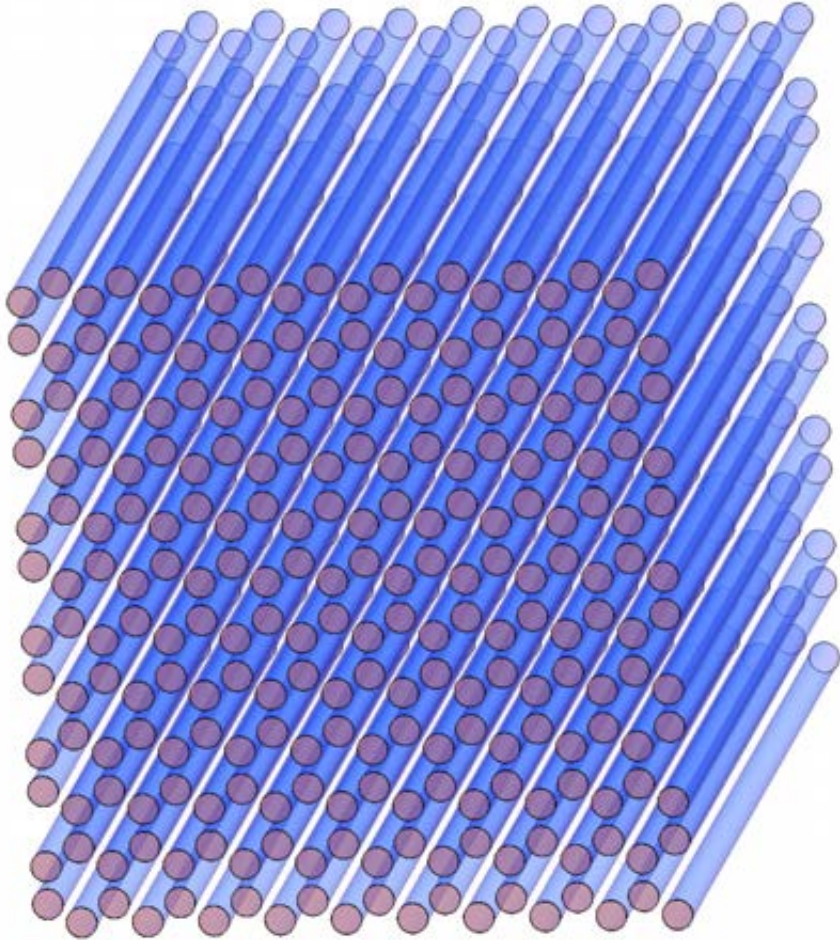
Our system: topological protection against transverse backscattering



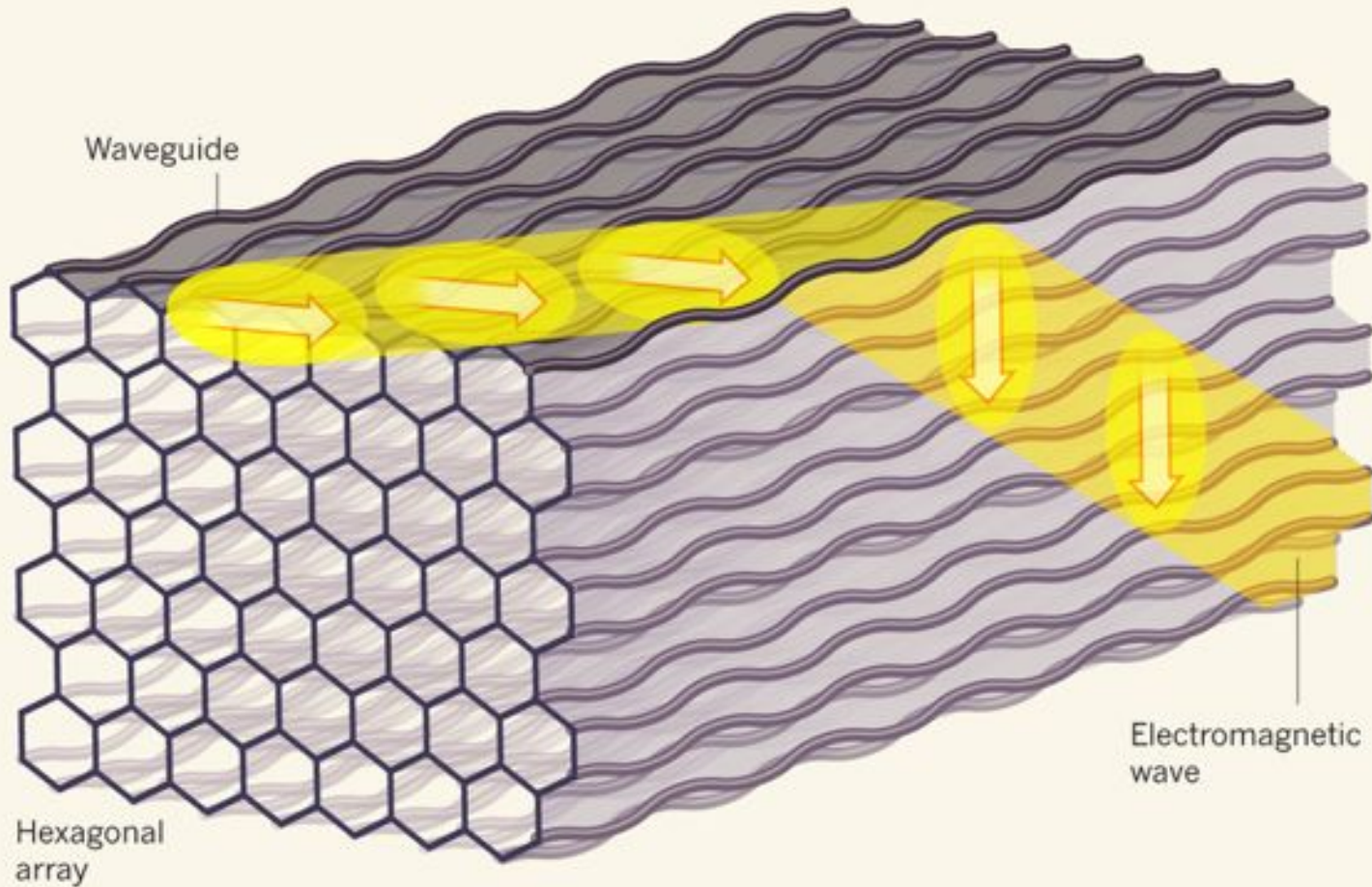
Graphene opens a Floquet gap for helical waveguides



Graphene opens a Floquet gap for helical waveguides

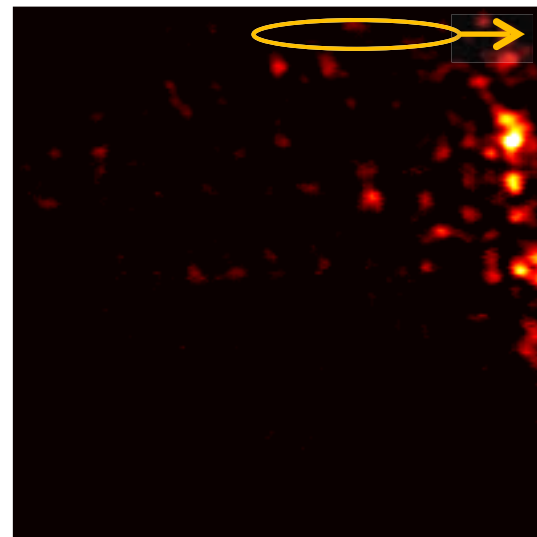
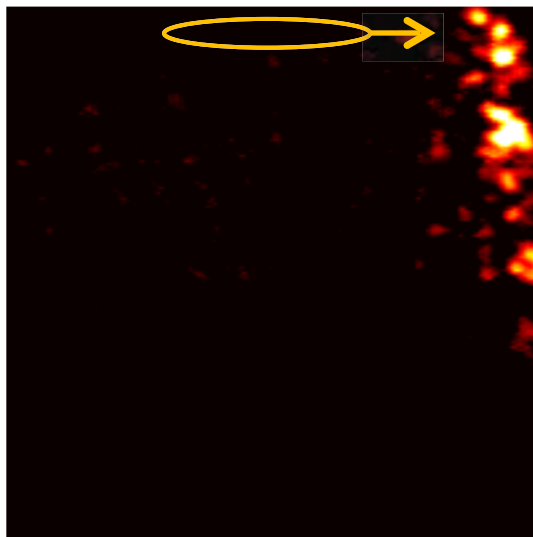
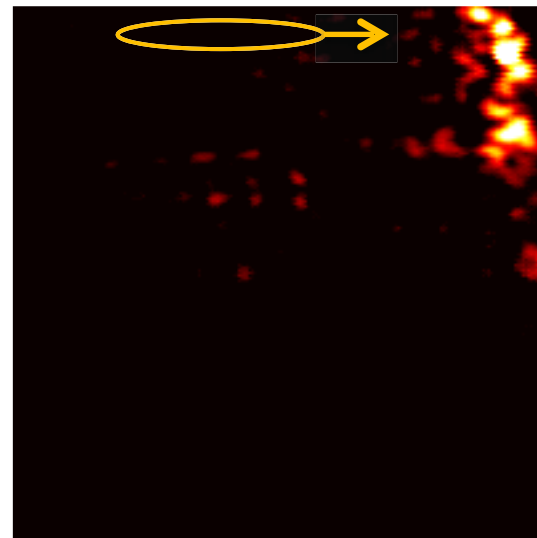
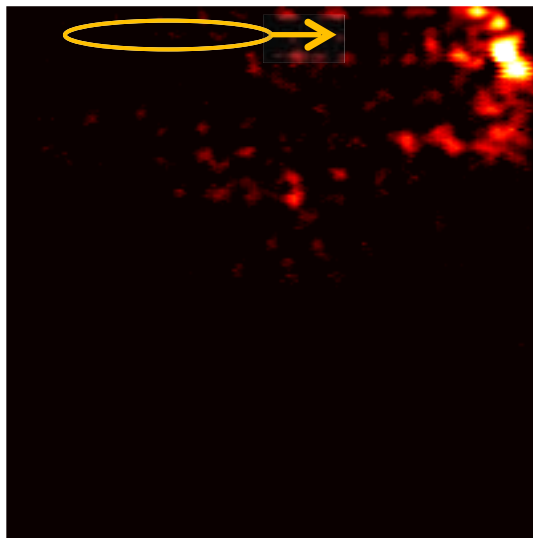
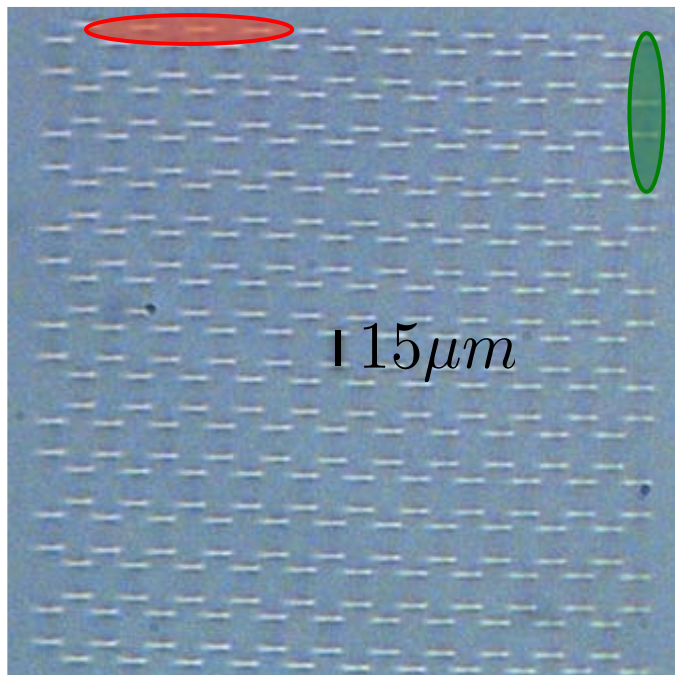


“Time”-domain continuous simulations

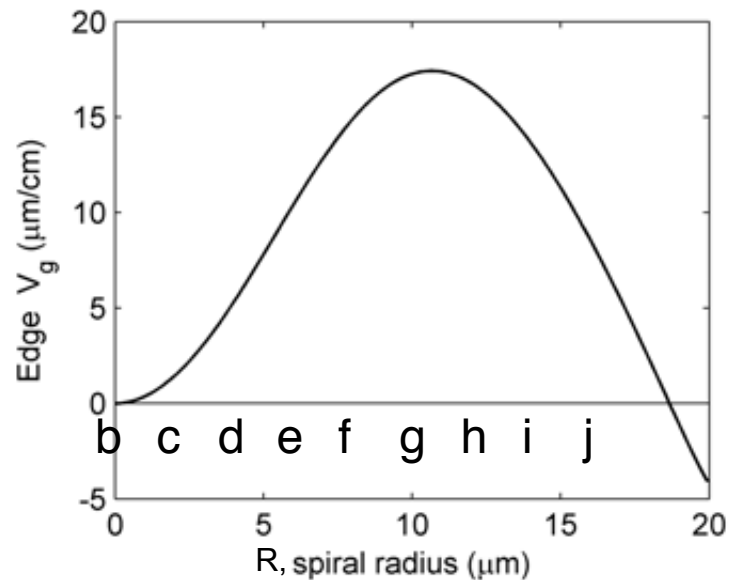
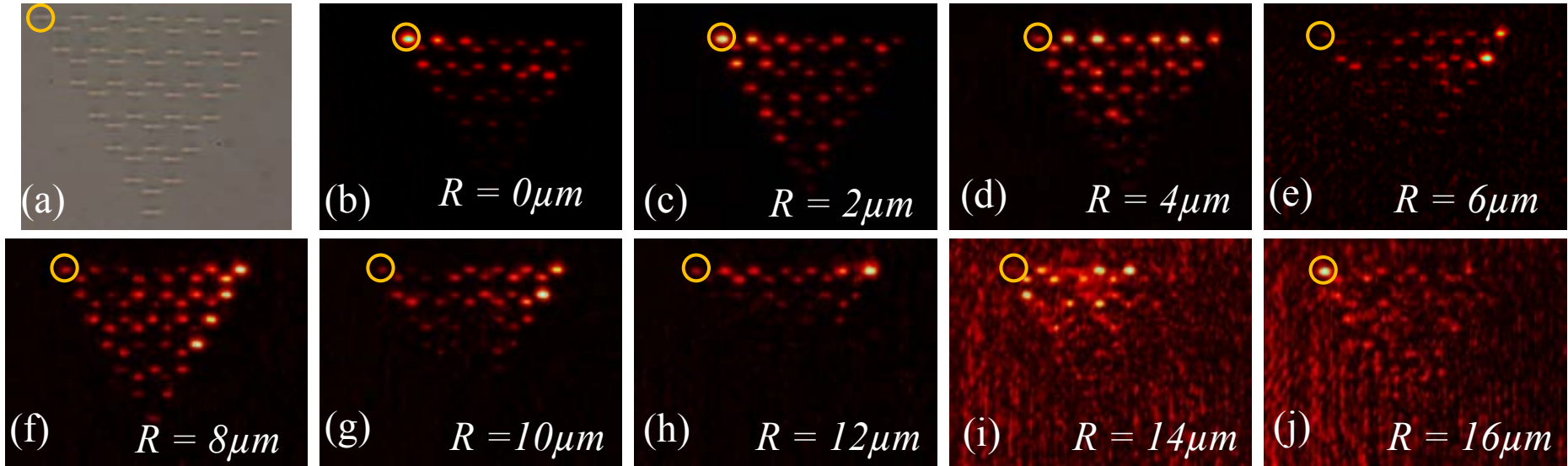


Experimental results: rectangular arrays

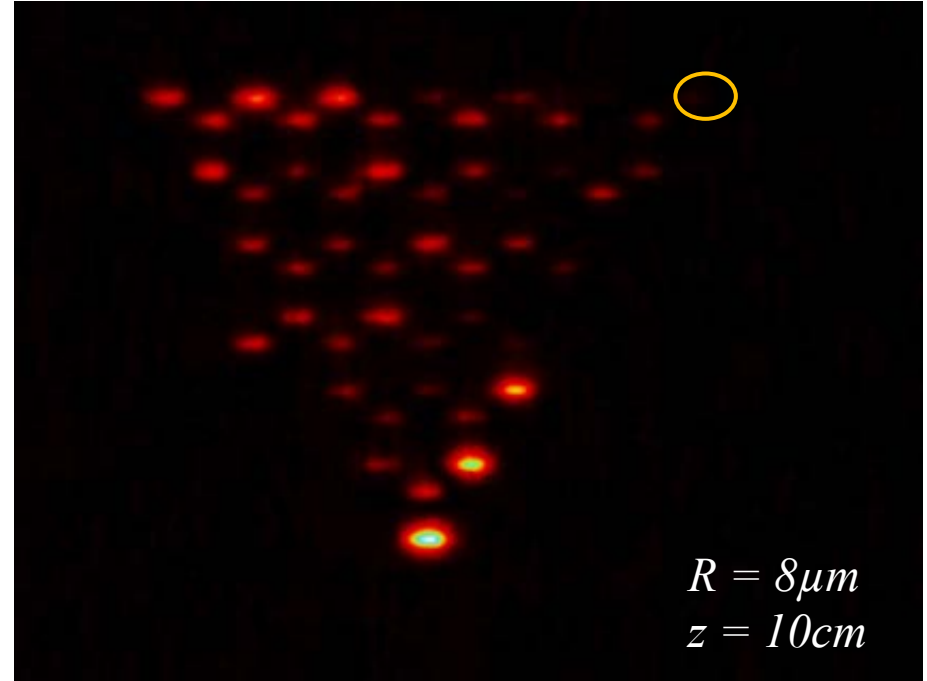
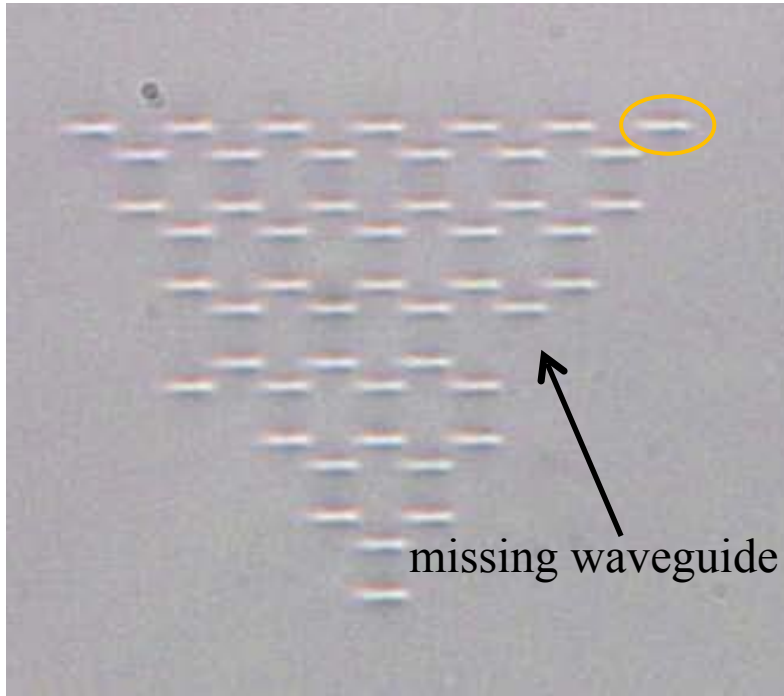
Microscope image



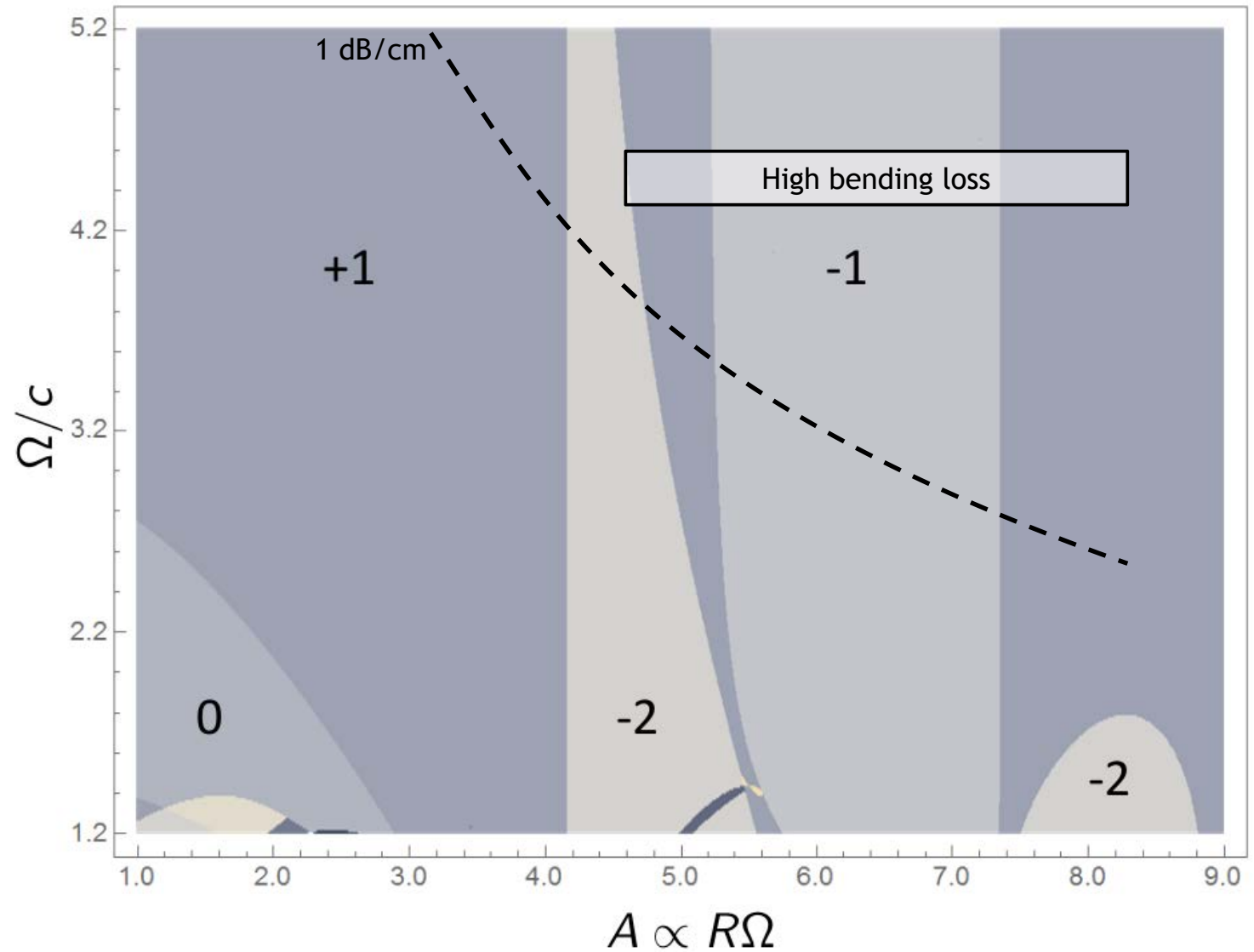
Experimental results: group velocity vs. helix radius, R



Experimental results: triangular arrays with defects



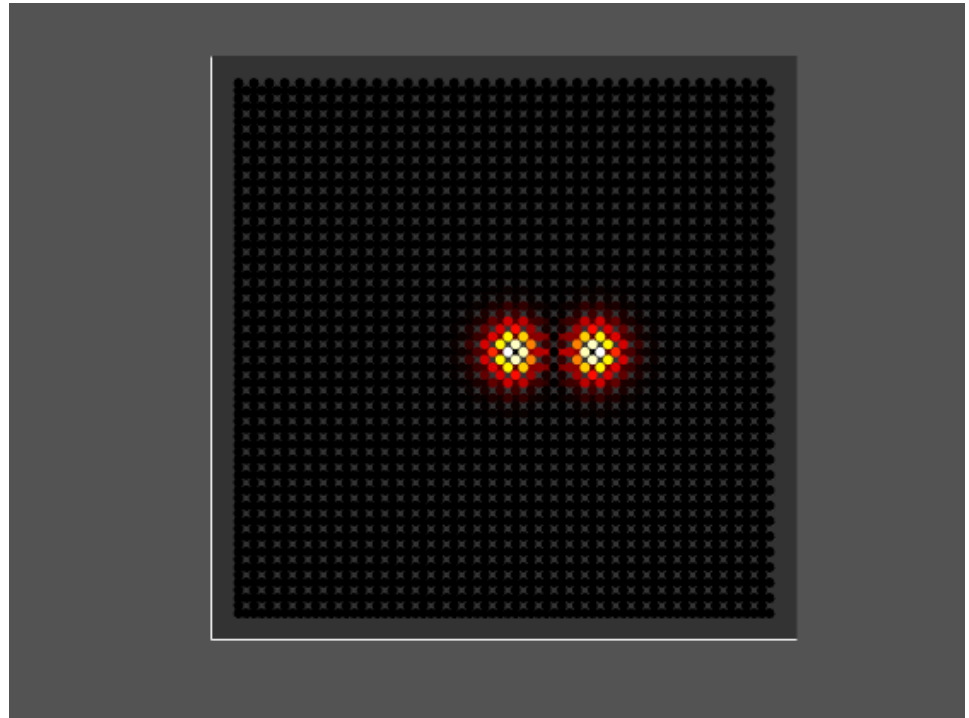
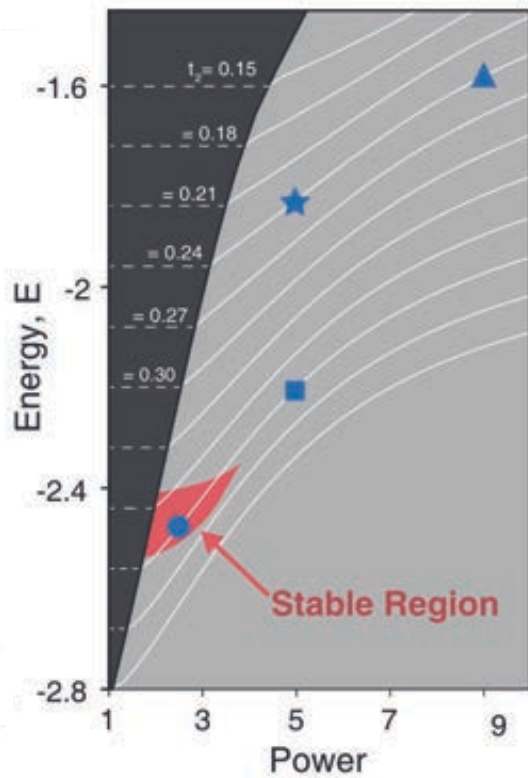
Observation of a topological transition



Interactions/nonlinearity: topological solitons

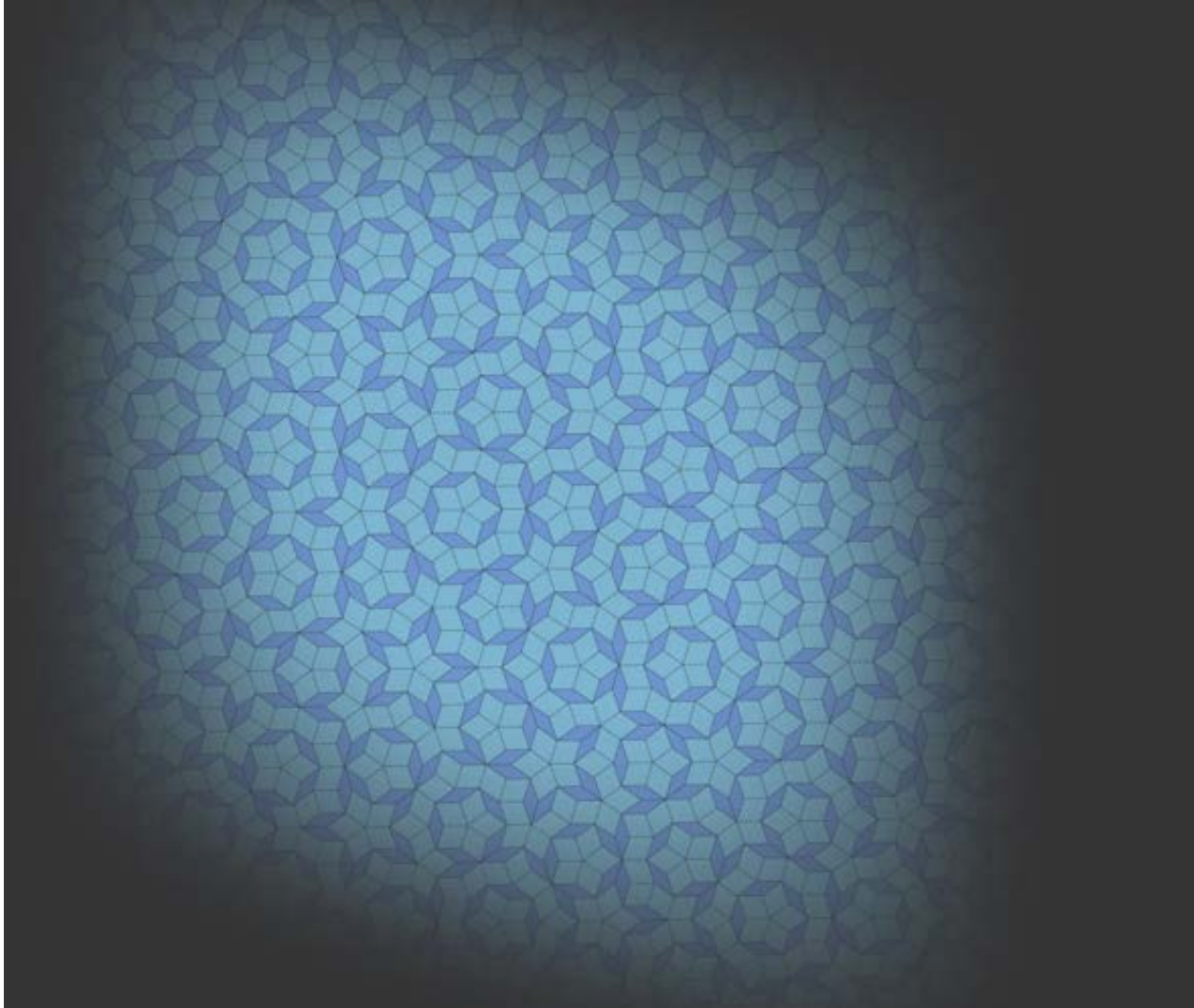
$$i\partial_z\psi = H_T\psi - |\psi|^2\psi$$

Superfluid like...



Topological Quasicrystals

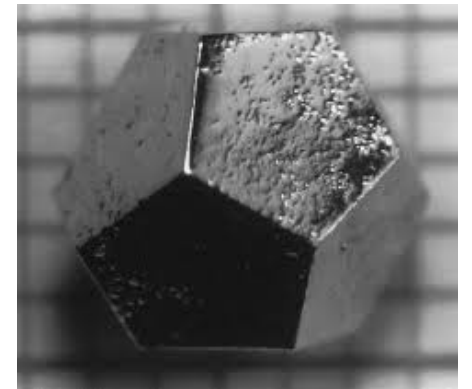
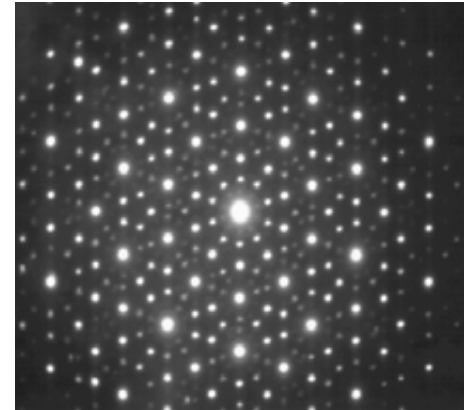
What are quasicrystals?



Dan Shechtman



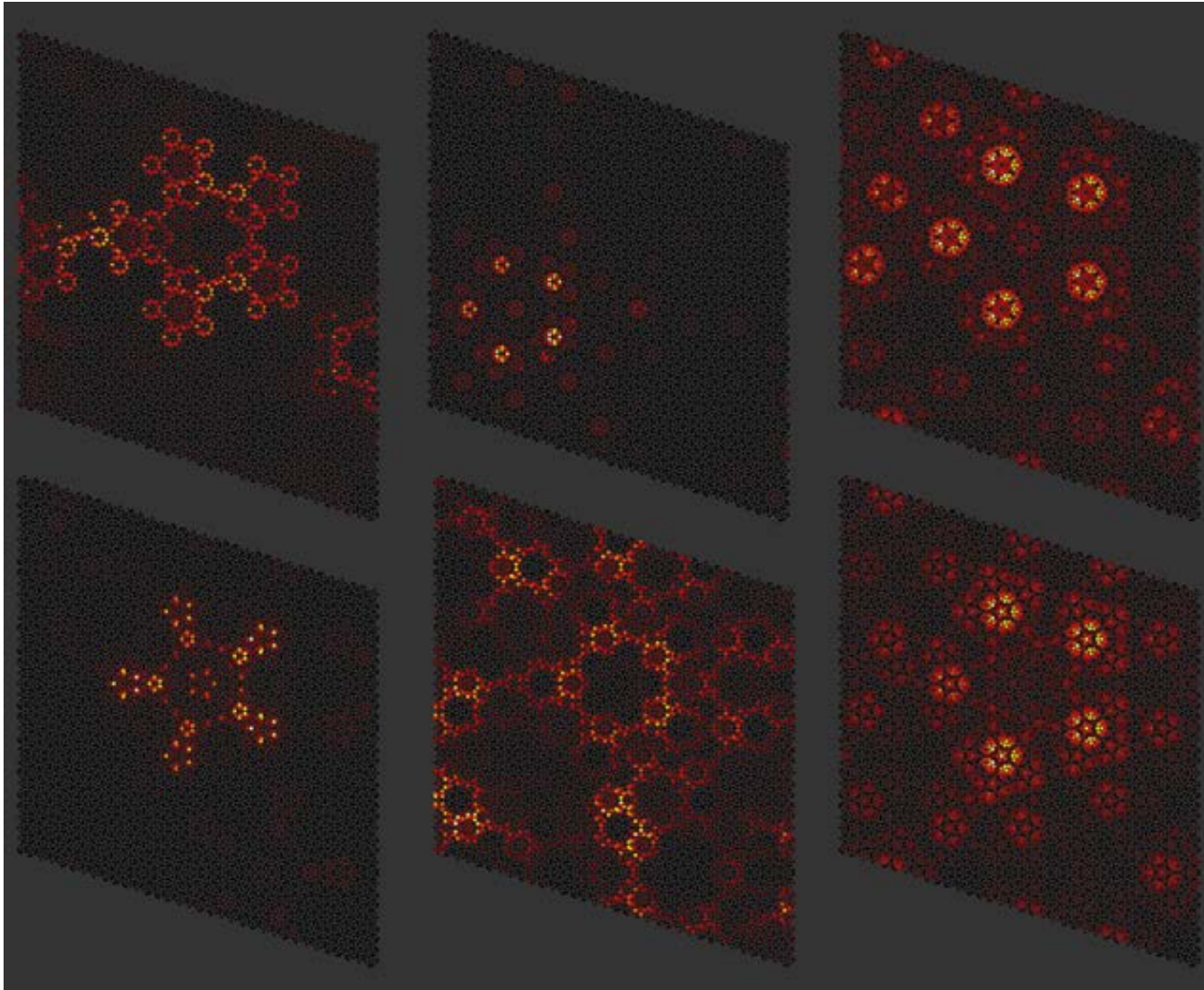
**Nobel Prize
2011**



Why study (photonic) quasicrystals?

- Fundamentally interesting: between disorder and periodicity; no k , no Bloch's theorem, rethink the nature of wave physics.
[Chan et al., *Phys. Rev. Lett.* **80**, 956-959 \(1998\); Tanese et al., PRL 112, 146404 \(2014\) .](#)
- Isotropic “Brillouin zone” means larger 2d gaps for low ϵ_2/ϵ_1 .
[Rechtsman et al., *Phys. Rev. Lett.* **101**, 073902 \(2008\).](#)
- Open question: do 3d photonic QCs have band gaps?
[Man et al., *Nature* **436**, 993-996 \(2005\).](#)
- Novel nonlinear behavior.
[Freedman, B. et al. *Nature* **440**, 1166-1169 \(2006\).](#)
- Surprising effects, e.g., disorder-enhanced transport.
[Levi et al., *Science* **332**, 1541 –1544 \(2011\).](#)

Quasicrystal bulk states

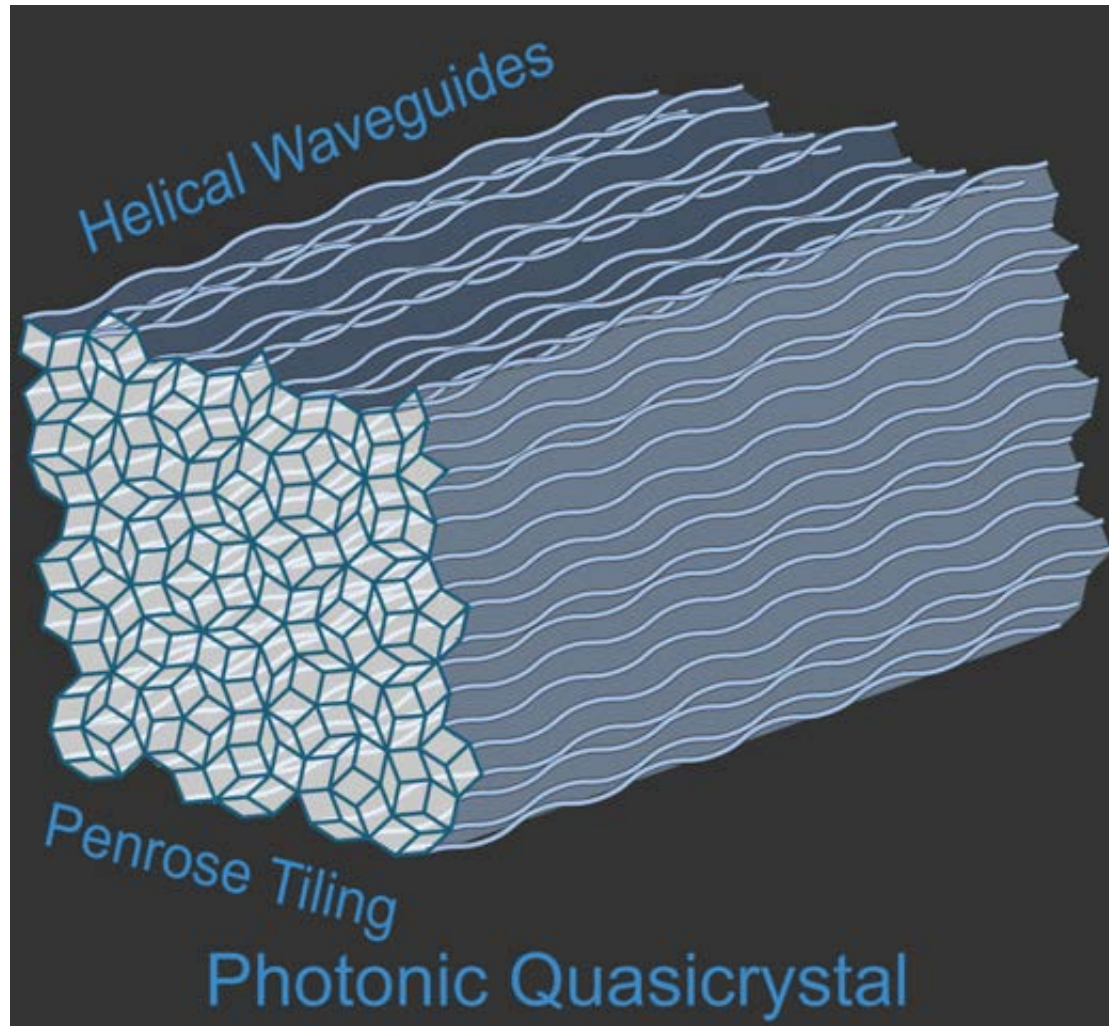


Fractal states

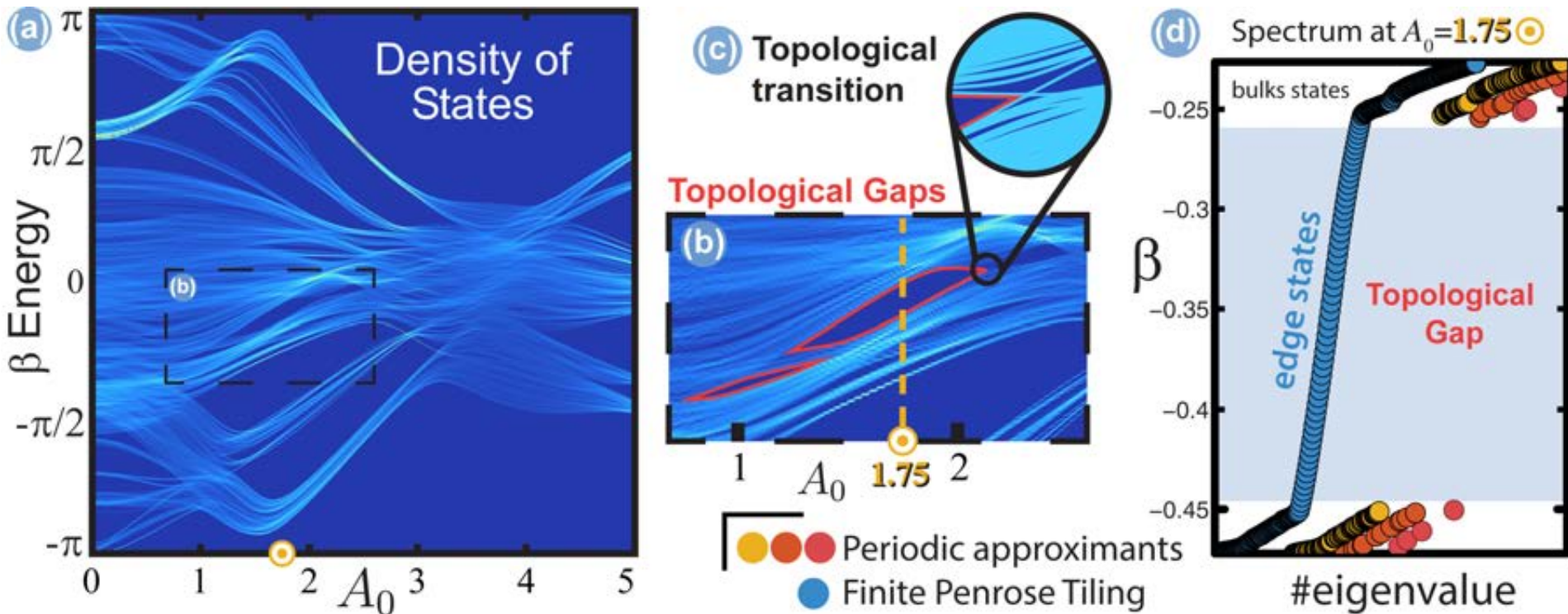
$$\psi \sim \frac{1}{r^n}$$

... strange
transport
properties

What happens in a Floquet'ed quasicrystal?



Topological gaps!

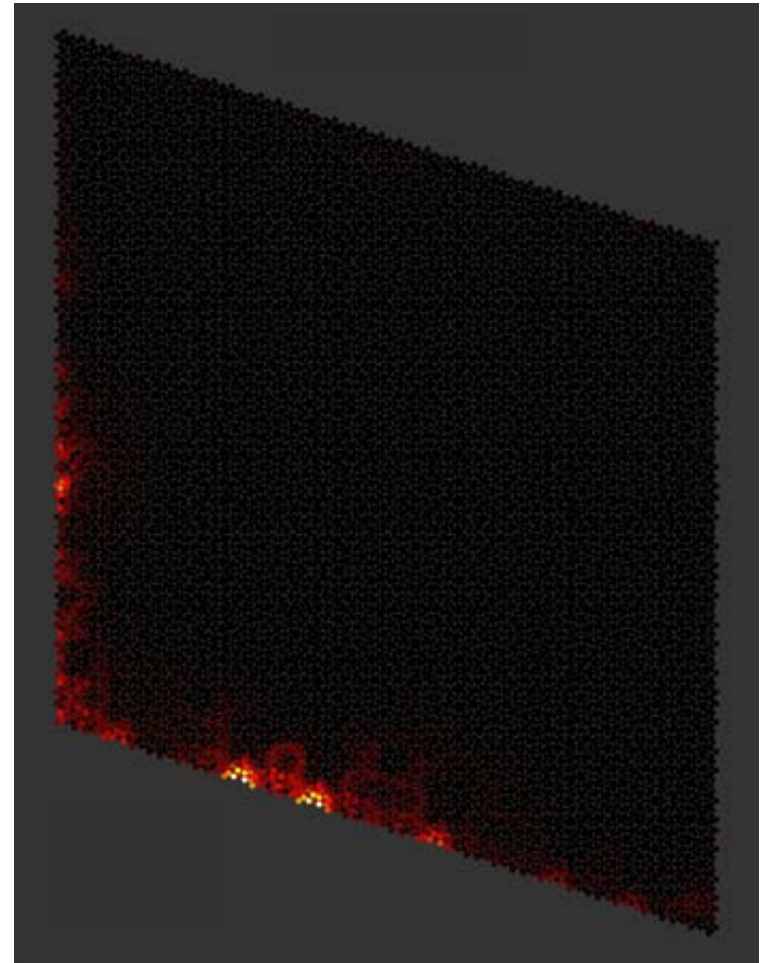
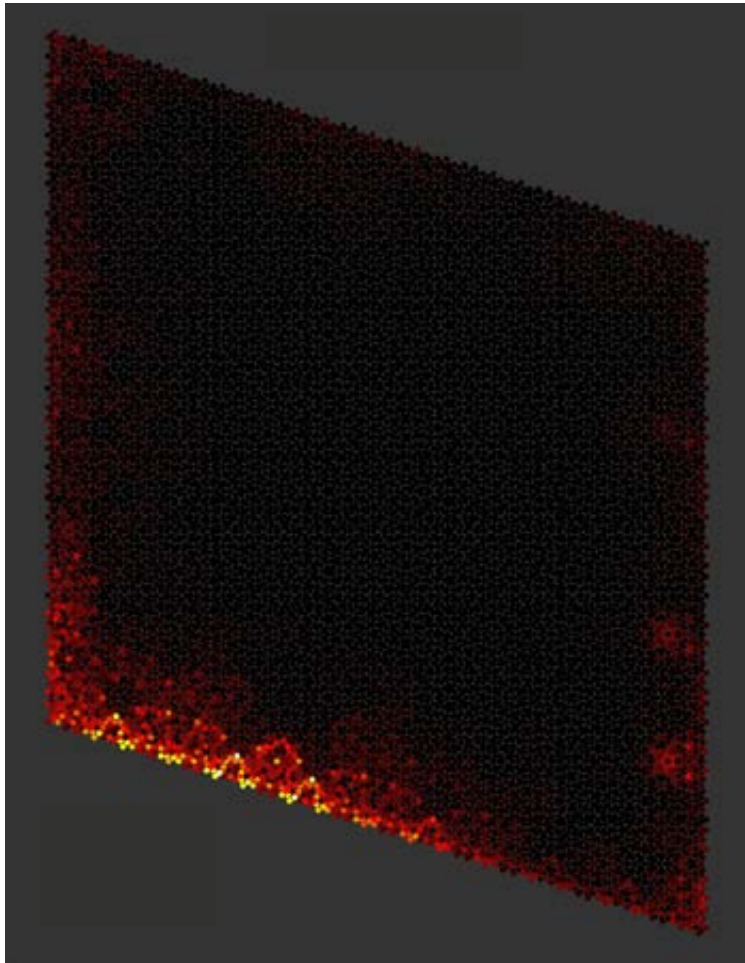


This is a quantum anomalous Hall effect (Haldane model) for quasicrystals!

→ new class of quasicrystalline states

Phys. Rev. X **6**, 011016 (2016)

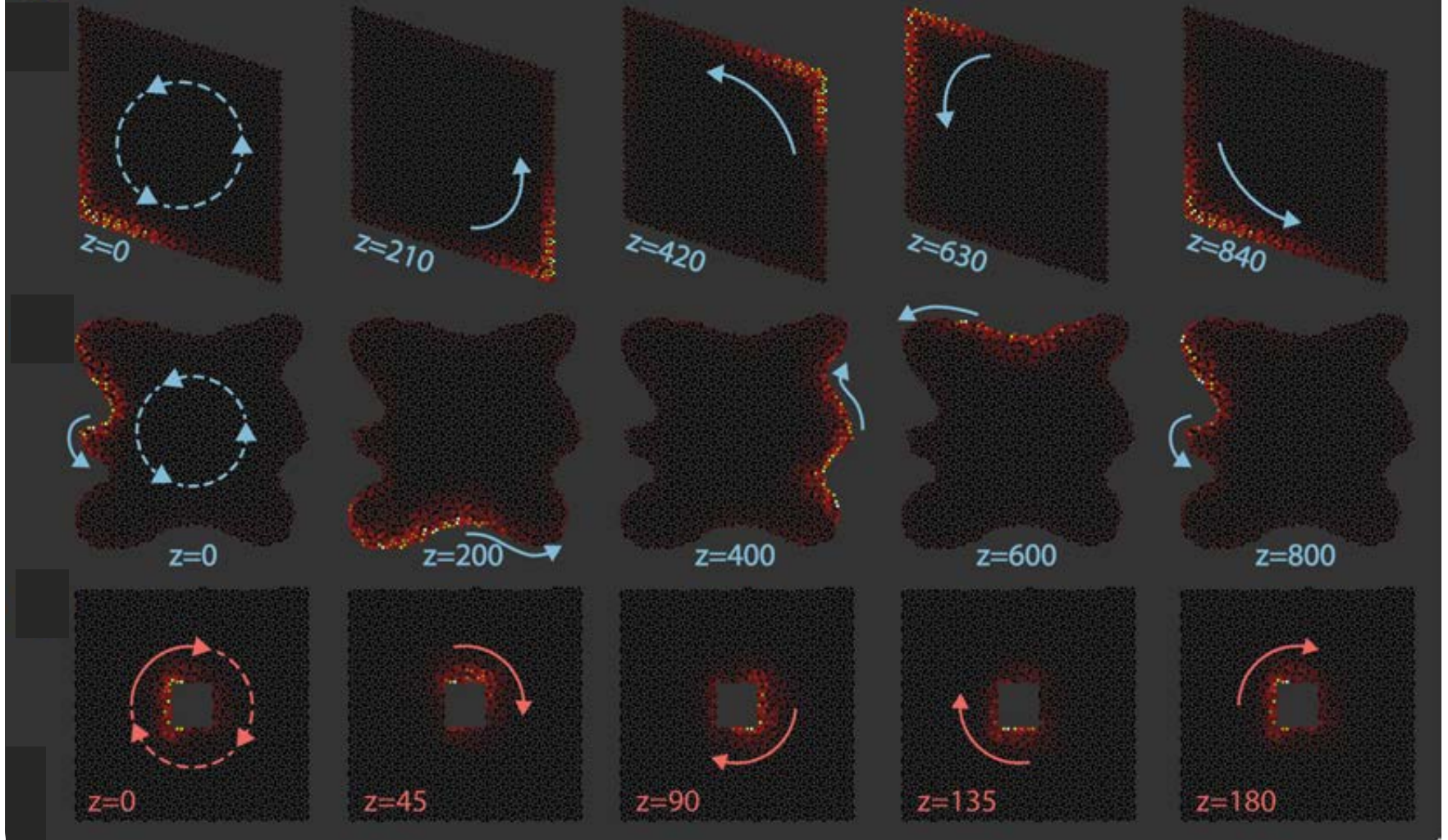
Topological edge states!



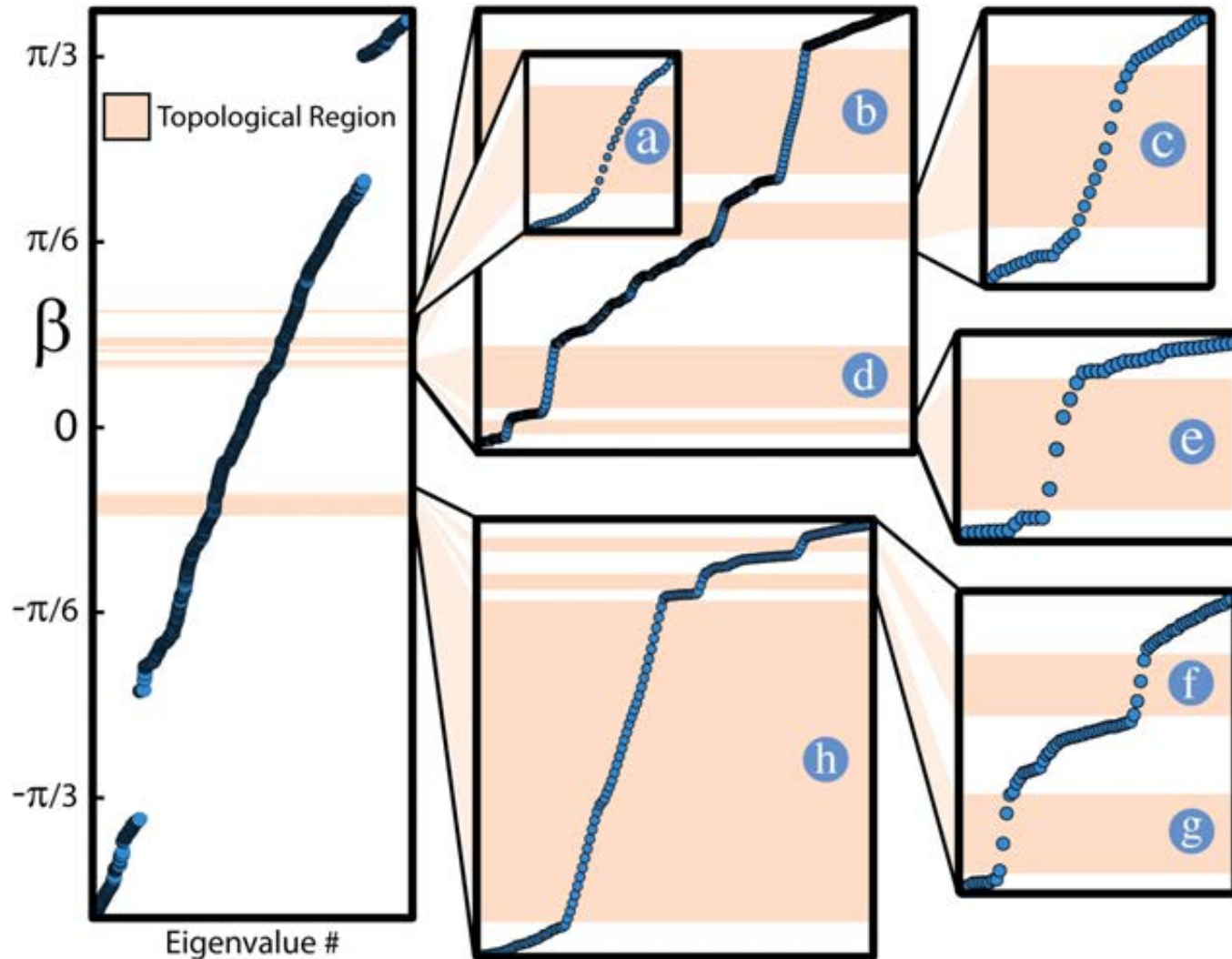
M. Bandres, MCR, M. Segev, PRX (2016)

Topological edge states!

Dynamics of Topological Edge Modes



Topological regions are fractal-like

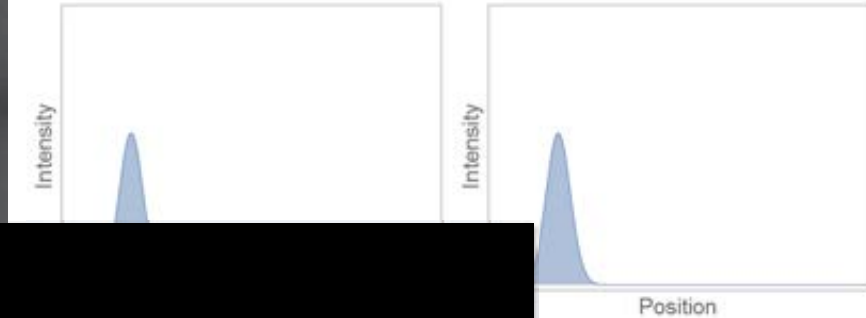
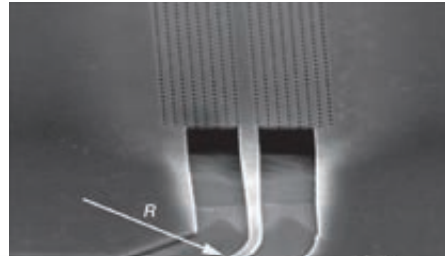
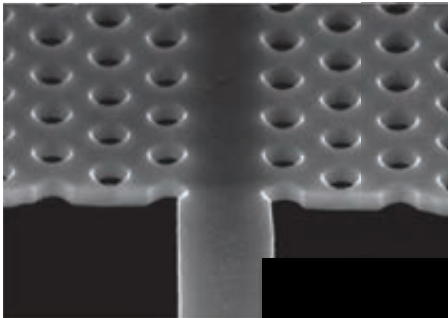


Conjecture: within any band, there are an infinite number of topological gaps

Topological slow light via BZ winding

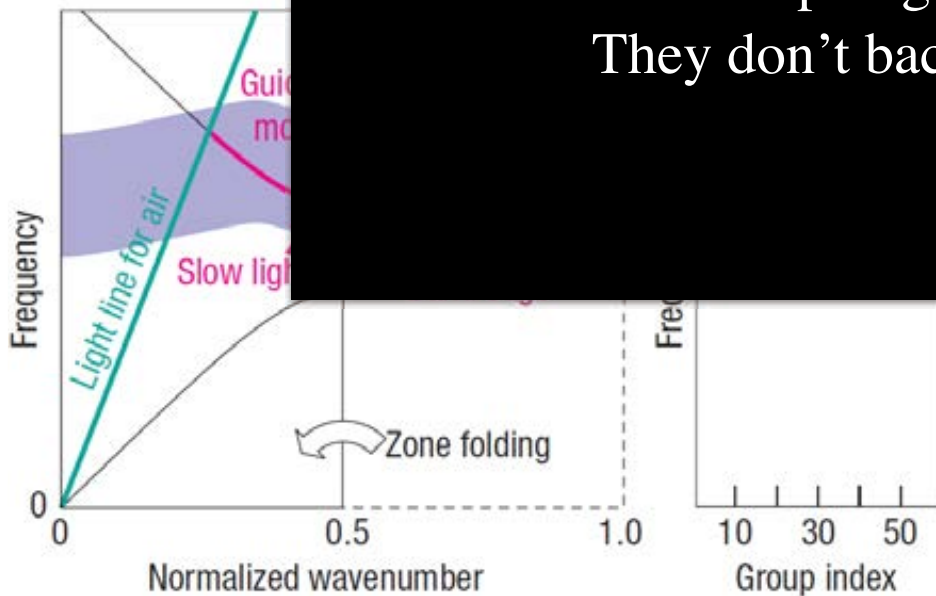
J. Guglielmon and M. C. Rechtsman, Phys. Rev. Lett. **122**, 153904 (2019).

Idea: topological slow light



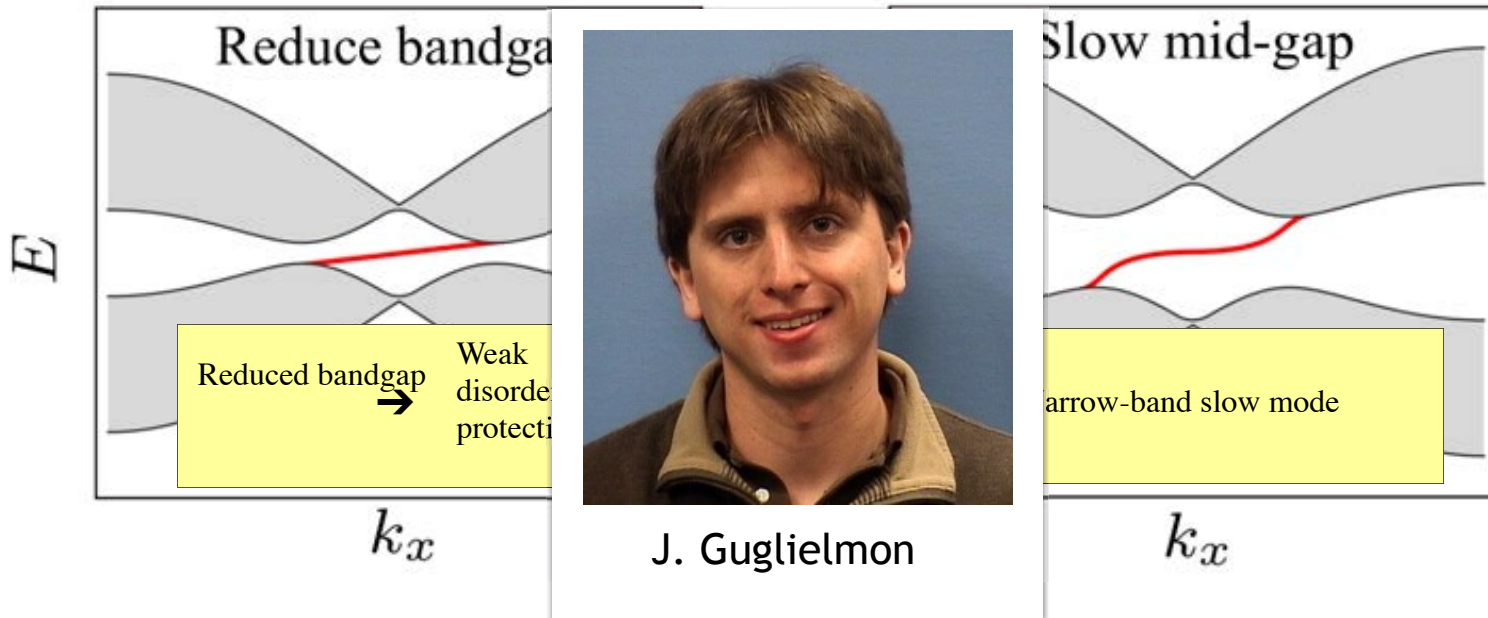
Vlasov, Yuri A.

Can we use topological edge states?
They don't backscatter...



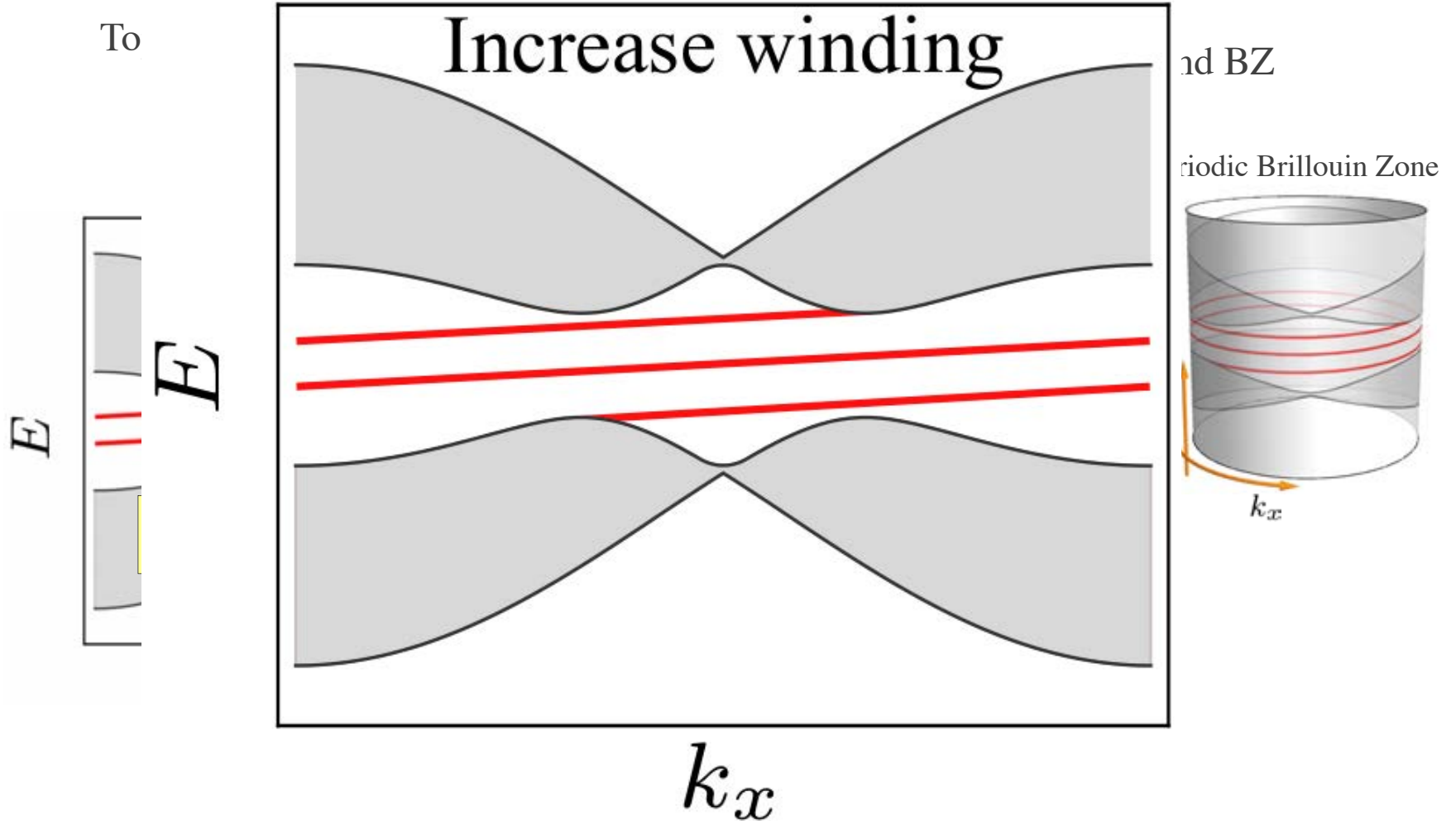
- Bandwidth
- Backscattering

Obvious ideas

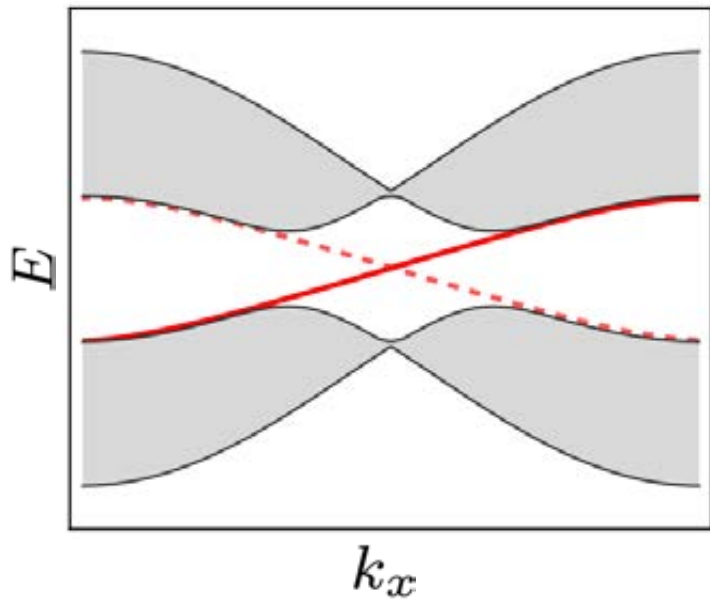
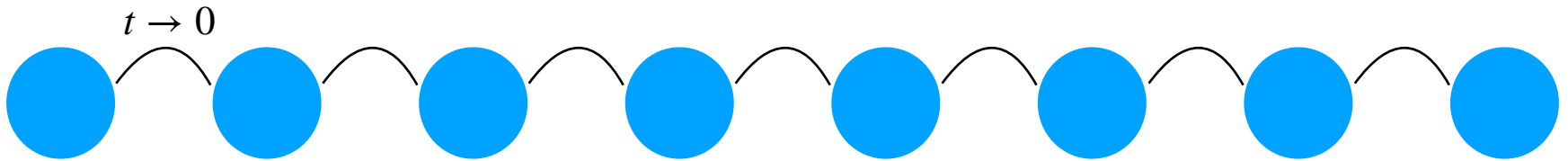


Note that both methods sacrifice bandwidth

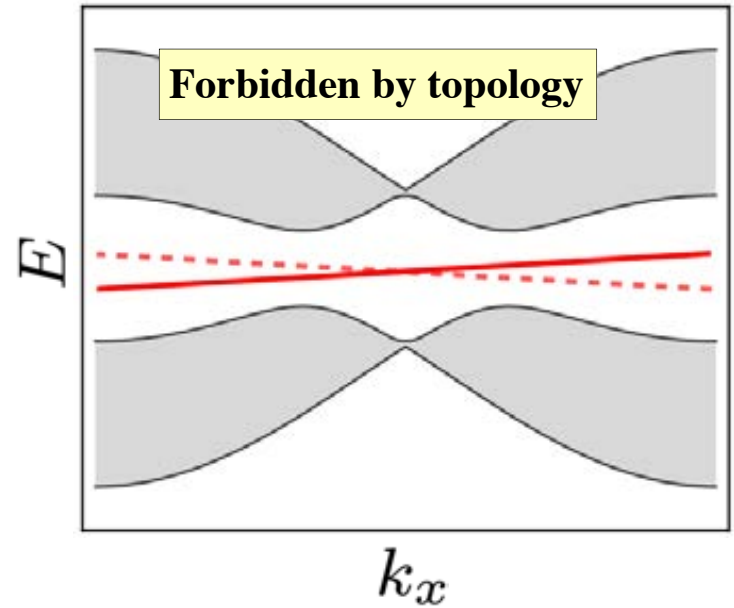
Increase winding



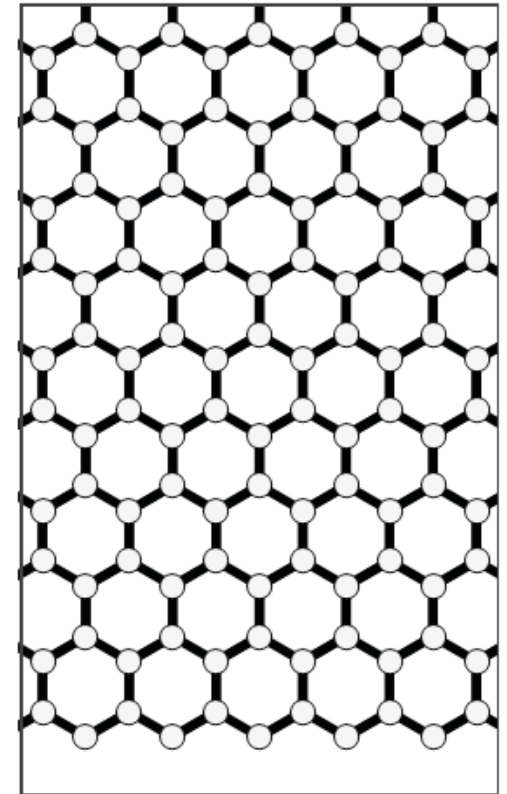
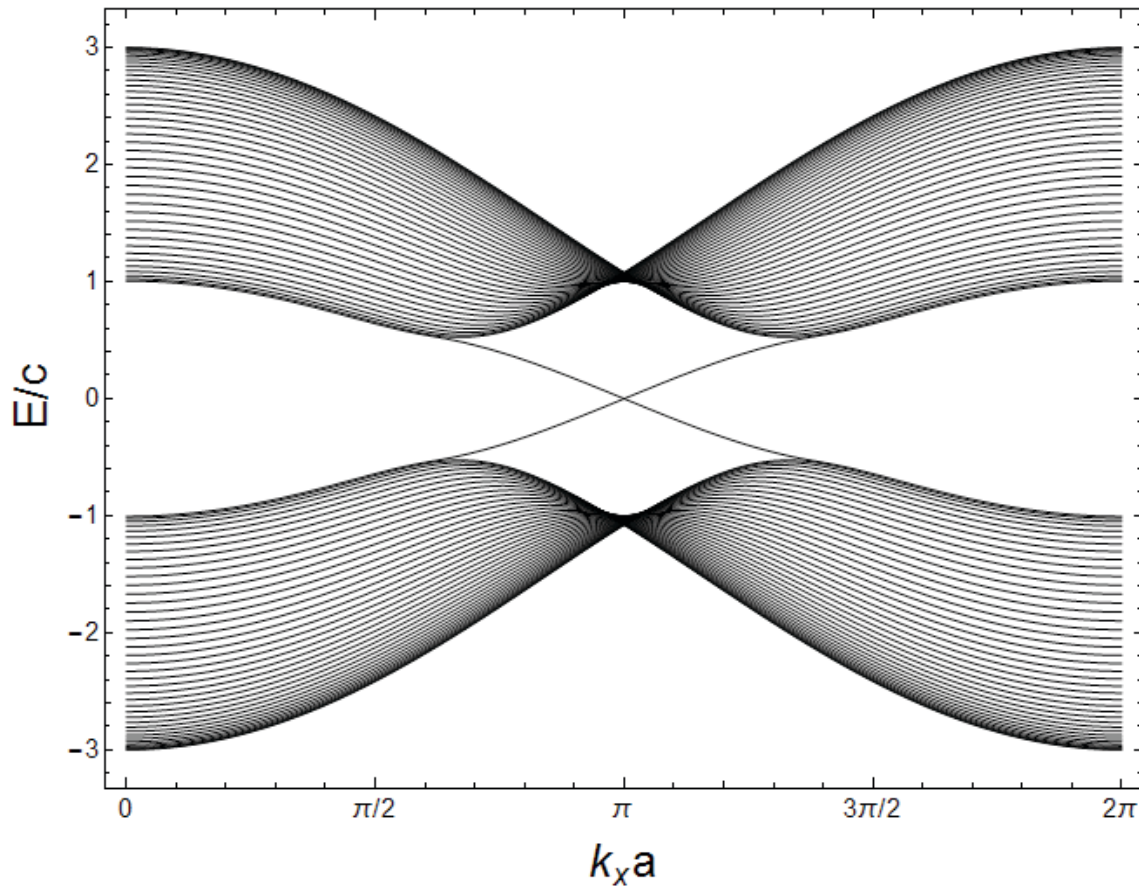
How do we do it?



Slow **all** in-gap
states



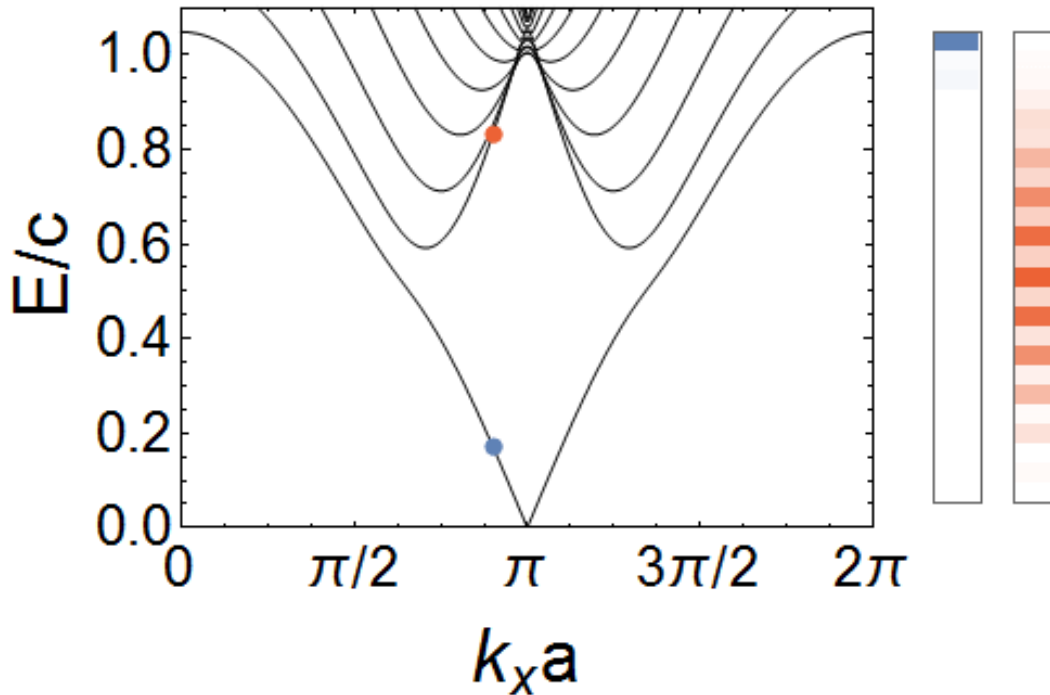
Engineering the edge...



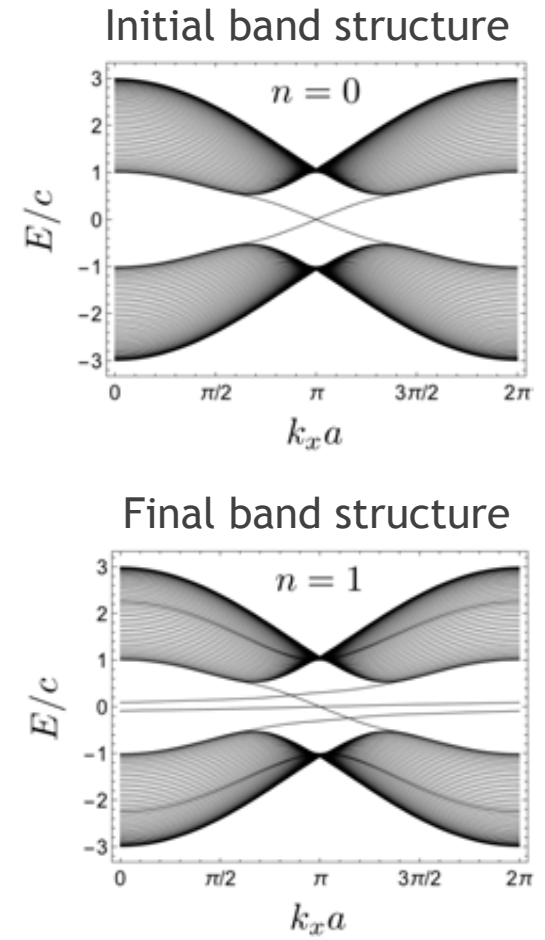
J. Guglielmon and M. C. Rechtsman. Phys. Rev. Lett. **122**, 153904 (2019).

Where do the new edge states come from?

$$H_\lambda(k_x) = (1 - \lambda)H_0(k_x) + \lambda H_1(k_x)$$

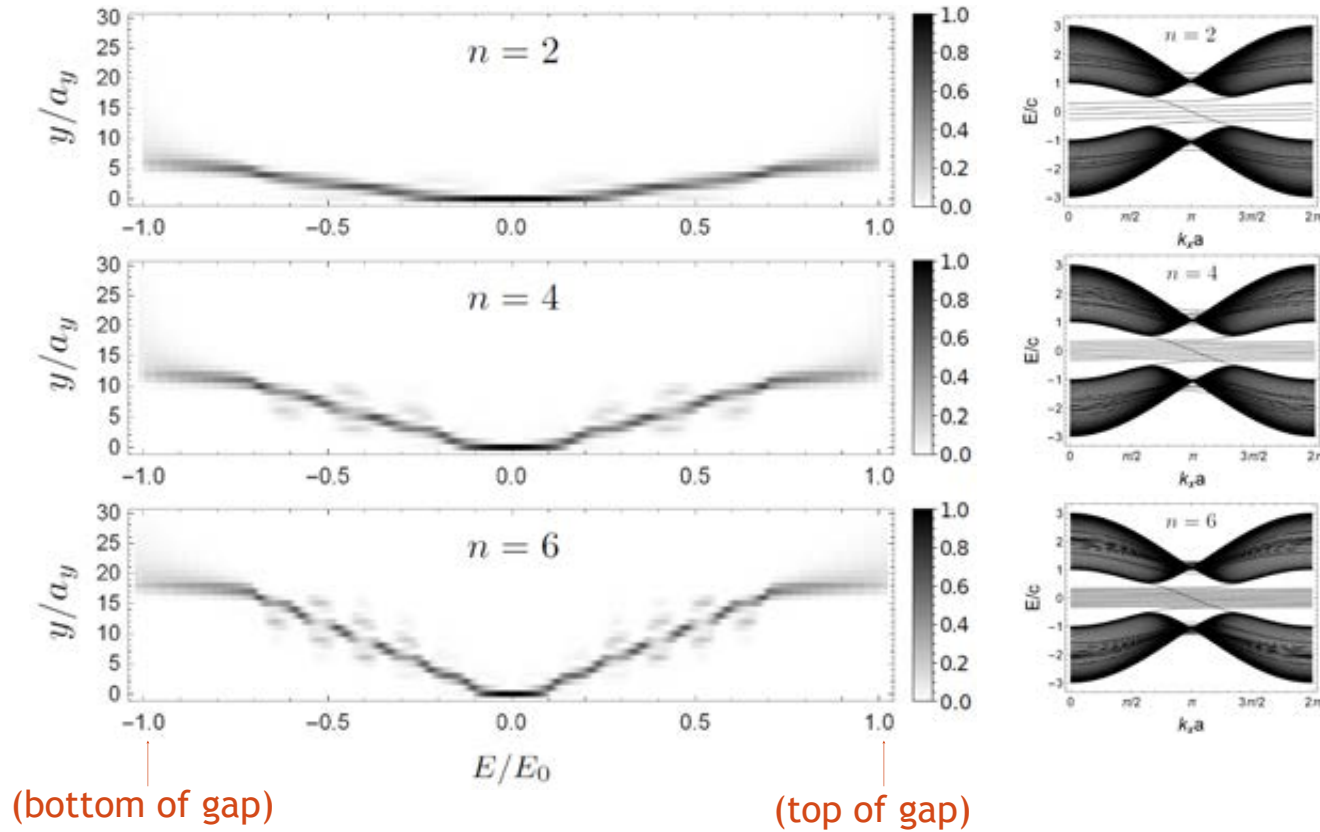


... this defines an invariant



Confinement of slow light edge modes

As winding increases, the edge modes utilize more bulk sites



“Unused real estate”
of a 2D PTI utilized to
enable wideband
operation

Robust slow light

These slow chiral edge states resist the severe backscattering associated with a reduced group velocity:

