Nanophotonics 2018- - Problem Set 1

Class Dates: *April 3 and April 5*
Due: *April 12*
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1 Snell's law from transverse wave vector conservation

Consider the refraction of a plane wave at a planar interface.

1. Using the boundary conditions at the interface, prove that the transverse wave vector $k_{||}$ must be conserved.
2. Derive Snell's law by using the invariance of the transverse wave vector.

2 Fresnel reflection coefficient (p-polarization)

Consider an interface between two media 1 and 2 with dielectric constants $\epsilon_1 = 2.25$ and $\epsilon_2 = 1$, respectively. The magnetic permeabilities are equal to one. A p-polarized plane wave with wavelength $\lambda = 532$ nm is incident from medium 1 at an angle of incidence of θ_1.

1. Derive the Fresnel reflection coefficient.
2. Express it in terms of amplitude A and phase Φ and plot A and Φ as a function of θ_1. What are the consequences for the reflected wave?

3 Evanescent field (I)

An evanescent field is an oscillating electric and/or magnetic field whose energy is spatially concentrated around a point or an interface. It can be described by plane waves ($\boldsymbol{E}e^{\mathrm{i}(\boldsymbol{kr}-\omega t)}$) with at least one component of the wavevector ($\boldsymbol{k}$) is imaginary. Total internal reflection is a typical way to generate an evanescent field.

1. A plane wave is incident on a water(n=1.33)-air interface at a 60° angle with the surface normal (z-axis). Show that in air, the z-component of the wavevector is imaginary.
2. Show that the z-component of the time-averaged Poynting vector $\langle S \rangle_z$ vanishes for this wave in air.
3. How far does the field extend in z? This field is frequently used in microscopy. Compare the spatial extent at a wavelength of 632 nm (HeNe laser) to the size of a bacterium. What does this tell you?

4 Evanescent field (II)

Here we will look at the polarization state of an evanescent field propagating in the x-direction created by total internal reflection of a p-polarized plane wave. The wave reflects of an interface at $z = 0$, where positive z is in the low index medium.

1. Calculate the time-dependent electric field $E_2(x,t) = (E_{2,x}(x,t), 0, E_{2,z}(x,t))$ just above the interface ($z = 10$nm).
2. For a fixed position x, the electric field vector $(E_{2,x}(x,t), 0, E_{2,z}(x,t))$ defines a curve in the (x,z)-plane as the time runs from 0 to λ/c. Determine and plot the shape of this curve. For numerical values choose $\theta_1 = 60°$, $\lambda = 600$ nm, $n_{down} = 1.5$, $n_{up} = 1$. Compare this shape to the shape defined by the field inside the high index medium. *Hint: plot these fields as a Lissajous figure using the parametric curve function, with time as the variable.*
3. There are certain fluorescent emitters that are sensitive to the handedness of circularly polarized light. These display so-called circular dichroism. If one would like to probe this effect with emitters at the above mentioned interface, on what side of the interface should one place the emitters? How could one switch the handedness of the circular polarization?