Nanophotonics 2018 - Problem Set Microscopy and LDOS

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Question 1: Far-field and near-field microscopy

Note: In addition to the class slides, the review article *Trends and developments in scanning near field optical microscopy* by V. Sandoghdar (in particular sections 2-3) can be of useful for this assignment.

Goal: To simulate and compare the image formation of a nanoscale object by an aberration-free conventional microscope and by a scanning near-field aperture.

Instructions: The electric field distribution in the object plane at (z = 0) is contained in `object2018.dat`, that can be downloaded from

`https://amolf.nl/research-groups/resonant-nanophotonics/uva-mastercourse`

It is a 400×400 pixel matrix that represents the field in a 8000×8000 nm area. The maximum intensity is 1, the minimum 0. The wavelength of illumination is 600 nm. You may use the Mathematica notebook provided, which contains some of the necessary functions. In your report, please describe how you calculate each part of the exercise.

The $N \times N$ matrix of the image contains elements a_{pq} that describe the field at and $x = (p-1)\Delta x$ and $y = (q-1)\Delta y$. Δx and Δy are $8\,\mu m/400\,\text{px} = 20\,\text{nm}$. We can perform a discrete two-dimensional Fourier transform of this matrix to obtain the angular spectrum representation.

$$\hat{a}_{uv} = \frac{1}{N^2} \sum_{p=1}^{N} \sum_{q=1}^{N} a_{pq} \exp\left[\frac{2\pi i}{N}\left(\left(u - 1 - \frac{N}{2}\right)(p-1) + \left(v - 1 - \frac{N}{2}\right)(q-1)\right)\right] \tag{1}$$

In the lecture, we saw that an analytical Fourier transform of the electric field is given by:

$$\hat{E}_{k_x,k_y;z} = \iint_{-\infty}^{+\infty} E(x,y,z) \exp\left[ik_x x + ik_y y\right] \mathrm{d}x\,\mathrm{d}y \tag{2}$$

1a: What is the relation between u and v and the quantities k_x and k_y?

1b: Calculate and plot the angular spectrum representation of the field in the plane of the object. Explain what you see in this image.

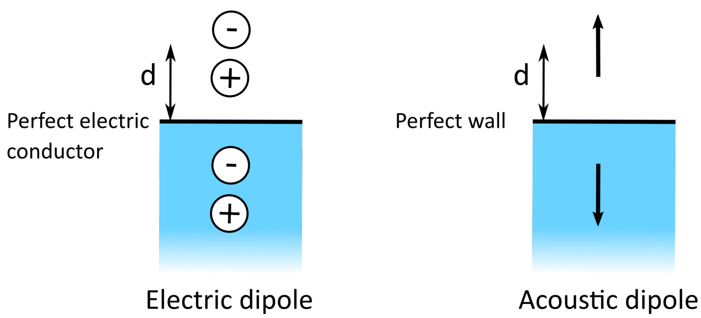
1c: Evanescent waves will not be collected by an objective. We will explore what this means for the image. As discussed in the lecture, high-k waves will become evanescent. Explain why this is. Simulate the intensity distribution in an image plane that is formed by an aberration-free lens system that collects all propagating anagles (i.e., a microscope objective with NA = 1). Plot this image. Explain what you see and how this is caused.

1d: A way to get around the diffraction limit is by using a scanning probe, for which an aperture is used. It consists of a circular hole with a diameter of 80 nm mounted in a square tip also with sides of 80 nm. The aperture is scanned at a constant height of z = 0 nm. Simulate the image formed when the aperture scans over the sample in steps of 20 nm. The amount of light scattered by the aperture only depends on the intensity under the aperture. Compare with your answers to 1c, and explain the result.

Question 2: A gong in front of a mirror

In the lecture we have seen how an electric dipole's radiation pattern is described when this electric dipole is placed in front of a mirror. In this exercise, we are going to see what happens when a gong, an acoustic dipole, is placed in front of a concrete wall.

When a gong is placed in front of a wall, two things should be considered: Firstly, while electric fields have a node at a mirror/wall, acoustic waves have an antinode. This results in opposite signs in their mirror images, as shown in the figure below. Secondly, the radiation pattern $S(\theta, \phi)$ is different for an acoustic dipole compared to an electric dipole.

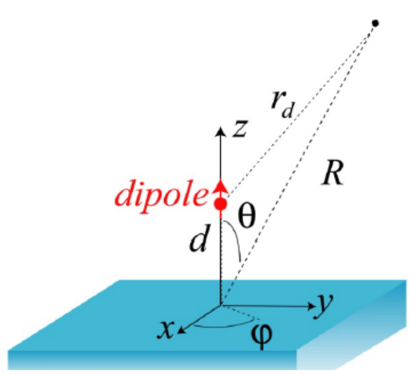


2a: For now, we will limit ourselves to dipoles perpendicular to the interface. What happens to the radiated power of the acoustic dipole for $\lim_{d\to 0}$? And for the electric dipole?

2b: In the presence of a wall the field at an observation point $\mathbf{R}$ with $R \gg \lambda$ reads:

$$\mathbf{U}(\mathbf{R}) \approx \frac{e^{ikR}}{R} S(\theta, \phi) \left[e^{ikd\cos\theta} + q\, e^{-ikd\cos\theta} \right] \qquad (3)$$

where $S(\theta, \phi)$ is the radiation pattern of an acoustic dipole and q represents the strength and sign of the mirror-image gong.



The amplitude of an acoustic pressure wave goes as

$$\mathbf{U}(\mathbf{R}) = S(\theta,\phi)\frac{e^{ikr}}{r}, \tag{4}$$

where $k = 2\pi/\lambda$, r is the distance to the observer, and $S(\theta,\phi)$ is the radiation pattern of the dipole. Derive Eq. 3 for an acoustic dipole at a distance d from a wall. First, show that $r_d \approx R(1 - \frac{d}{R}\cos\theta)$ is valid at large distances from the wall, $R \gg d$. Hint: Taylor expand r_d, and use that the system is spherically symmetric around $\hat{z}$. Refer to the figure below for the geometry.

The radiated flux of a dipole scales with $|U(R)|^2$. For acoustic dipoles, the radiation pattern is shaped as $A\cos\theta$.

2c: Write down and simplify the integral for the radiated flux by the acoustic dipole as far as possible. You can use standard Bessel function integrals or Mathematica for the integral itself. Hints: Do you need to integrate over a full sphere?

Do the same for an electric dipole. These also follow Eqn. 3, but the radiation pattern $S(\theta,\phi)$ for electric dipoles is shaped as $B\sin\theta$.

Plot the radiated power as a function of d/λ, normalized to the radiated power of an acoustic/electric dipole in free space. Compare and explain the behaviour of the acoustic and electric dipoles.

2d: Your result for the change in emitted power of a classical point dipole is equivalent to the change of the spontaneous emission decay rate of a quantum dipole. It tells you that depending on the geometry, an atom can decay faster or slower due to constructive or destructive interference with its own mirror image. This is called the local density of optical states (LDOS). Consider the diagram you drew in the previous question. Do you expect the LDOS of the horizontally oriented dipole to be enhanced or reduced for $\lim_{d\to 0}$? Explain why. What happens at large distances d?