Photonic crystals

Semi-conductor crystals for light

The smallest dielectric – lossless structures to control whereto and how fast light flows

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Definition:

A photonic crystal is a periodic arrangement with a $\sim \lambda$ of a dielectric material that exhibits strong interaction with light - large $\Delta \varepsilon$. 

![Diagram of photonic crystals](image)
Bragg diffraction

Bragg mirror
Antireflection coatings (Fresnel equations)

Bragg’s law:

$$2n_{\text{average}} d \cos(\theta) = m \lambda$$

Each successive layer gives phase-shifted partial reflection

Interference at Bragg’ condition yields 100% reflection
Dielectric mirror

R arbitrarily close to 100%, independent of index contrast (N-1) end-facet Fabry Perot fringes
Wave vector $\mathbf{K}$ is equal to $\mathbf{K} + m2\pi/a$

“Bloch theorem”
Defects trap light

1D stack calculation

N layers, defect [thicker high index layer], again N layers

Localized resonance caught between mirrors
"Defect state" compare semiconductor donors & acceptors
2D and 3D lattices

- Dielectric constant is periodic on a 2D/3D lattice

$$\epsilon(r) = \epsilon(r + R) \quad \forall R = ma_1 + pa_2 + qa_3$$

Bravais lattices
- 2D: square, hexagonal, rectangular, oblique, rhombic
- 3D: sc, fcc, bcc, etc (14 x)
Reciprocal lattice

\[ \epsilon(r) = \epsilon(r + R) \]

\[ \epsilon(r) = \sum_{G} \epsilon_{G} e^{iG \cdot r} \]

\[ G = mb_1 + pb_2 + qb_3 \]

Reciprocal lattice with property

\[ e^{iG \cdot R} = 1 \]

Special wave vectors of scale \( b \sim 2\pi/a \)
Example of reciprocal lattice vectors

\[ b_1 = 2\pi \frac{a_2 \times a_3}{a_1 \cdot (a_2 \times a_3)} \]

\[ b_2 = 2\pi \frac{a_3 \times a_1}{a_2 \cdot (a_3 \times a_1)} \]

\[ b_3 = 2\pi \frac{a_1 \times a_2}{a_3 \cdot (a_1 \times a_2)} \]

Note how:

\[ b_n \cdot a_{m\neq n} = 0 \]

\[ b_n \cdot a_n = 2\pi \]
Suppose we look for solutions of:

\[ \nabla \times \nabla \times \mathbf{E} = \epsilon(r) \frac{\omega^2}{c^2} \mathbf{E} \]

with periodic $\epsilon(r)$

Bloch’s theorem says the solution must be invariant up to a phase factor when translating over a lattice vector.

\[ E_{n,k}(r + R) = e^{ik \cdot R} U_{n,k}(r) \]

Equivalent:

\[ E_{n,k}(r) = \sum_G U^n_{k,G} e^{i(k+G) \cdot r} \]

Phase factor  Truly periodic

1. Note how $k$ and $k+G$ are really the same wave vector
2. Note how this “$k$” is exactly the $K$ in the 1D eigenvalue $e^{iKd}$
Note how $k$ and $k+G$ are really the *same* wave vector

\[ E_{n,k}(r) = \sum \hat{U}_{k,G}^n e^{i(k+G) \cdot r} \]

‘Band structure’
Folding bands of 1D system

Bloch wave with wave vector \( k \) is equal to Bloch wave with wave vector \( k + m \frac{2\pi}{a} \)
Formal derivation in 3D

Wave eq:
\[ \nabla \times \left( \frac{1}{\varepsilon(r)} \nabla \times H(r) \right) = \frac{\omega^2}{c^2} H(r). \]

Bloch:
\[ \eta(r) = \frac{1}{\varepsilon(r)} = \sum_G \eta_G e^{iG \cdot r} \quad \text{and} \quad H_{n,k}(r) = \sum_G u_{G}^{n,k} e^{i(k+G) \cdot r}. \]

Substitute and find:
\[ \sum_G \eta_{G-G'}(k+G') \times [(k+G) \times u_{G}^{n,k}] = \frac{\omega_n(k)^2}{c^2} u_{G'}^{n,k} \quad \forall G' \]

The structure is that of an infinite dimensional linear problem frequency acts as the eigenvalue
\[ \mathbf{k} \times \mathbf{k} / \eta_0 \]

\[ \left[ \mathbf{k} + \mathbf{G}_1 \right] \times \left[ \mathbf{k} + \mathbf{G}_1 \right] / \eta_0 \]

\[ \begin{pmatrix} \mathbf{u} \\ \end{pmatrix} = \frac{\omega^2}{c^2} \begin{pmatrix} \mathbf{u} \end{pmatrix} \]
Dispersion relation of vacuum -folded

Folded “Free dispersion relation”
\[ \mathbf{k} \times \mathbf{k} / \eta_0 \]

\[ \left[ \mathbf{k} + \mathbf{G}_1 \right] \times \left[ \mathbf{k} + \mathbf{G}_1 \right] / \eta_0 \]

\[ \left[ \mathbf{k} \right] \times \left[ \mathbf{k} + \mathbf{G}_1 \right] / \eta_{\mathbf{G}_1} \]

\[
\begin{pmatrix}
\mathbf{u}
\end{pmatrix}
= \frac{\omega^2}{c^2}
\begin{pmatrix}
\mathbf{u}
\end{pmatrix}
\]
Nearly free dispersion

Crossing \rightarrow \text{ anticrossing upon off-diagonal coupling}

Compare QM: degeneracies are lifted by perturbation
More complicated example

FCC crystal
close packed
connected spheres

1st Brillouin zone (bcc cell)
Wedge: irreducible part
Folded bands – almost of vacuum

Example: \( n=1.5 \) spheres, (26% air)

1\text{st} Brillouin zone (bcc cell)

Wedge: irreducible part
Folded bands – almost of vacuum

Example: $n=1.5$ spheres, (26% air)

- Spaghetti
- Diffractive / Photonic crystal
- Effective medium / metamaterial
Folded bands – almost of vacuum

Example: \( n=1.5 \) spheres, (26\% air)

\[
2n_{\text{average}}d\cos(\theta) = m\lambda
\]

1/\( \cos(\theta) \) shift

Bragg gap at normal incidence to 111 planes
Wider bands

Reversing air & glass to reduce the mean epsilon

Note (1): band shifts up – lower effective index
(2): Relative gap broadens
Replacing $n=1.5$ by $n=3.5$, keeping $\sim 80\%$ air `airholes’
A true band gap FOR ALL wave vectors opens up
We counted *all the eigenstates* in the 1\textsuperscript{st} Brillouin zone. Note (1) a true gap, and (2) regimes of very high state density.
Si 3D photonic crystals

1. Colloids stack in fcc crystals
2. Silicon infilling with CVD
3. Remove spheres

Vlasov/Norris Nature 2001
Technique pioneered at UvA
(Vos & Lagendijk, 1998)
Woodpile crystals

Silicon – repeated stacking, folding
*Sandia, Kyoto*

GaAs (also $n=3.5$) – robotics in SEM
2D crystals

Si or GaAs membranes
Very thin (200 nm)
Kyoto, DTU, Wurzburg,...
Dielectric rod structure
TM means E out of plane

Snapshots of field at band edges 
\( k = \frac{G}{2} \) for \( G=X = \frac{2\pi}{a(1,0)} \)
\( G=M = \frac{2\pi}{a(1,1)} \)

Note the phase increment due to \( k \)

Note field concentration in air (band 2) 
rod (band 1)
Why all the effort for just a lot of math?

What we have seen so far:
• Photonic crystals diffract light, just like X-ray diffraction
• Unlike X-ray diffraction, the bandwidth is \( \sim 20\%\), not \( 10^{-4}\)
• Light has a nontrivial band structure

What is so great:
• A nontrivial band structure means control over how \textbf{fast} light travels, and how it refracts
• A true band gap expels \textit{all} modes
  • Complete shielding against radiative processes
  • Line and point defects would be completely shielded traps for light
1. In a 2D crystal polarization splits into TE and TM

2. Band structure is for in-plane k-only

3. ‘Light-line’ separates bound from leaky
A single line surrounded by a full band gap guides light. The band gap forbids it from escaping \textit{into} the crystal.
Line defects and bends

Exceptionally tight – 90° bend
Potential for very small low-loss chips
Measurement of guiding & bending

Sample: AIST Japan
Meas: AMOLF
Cavity in experiment

Free standing GaAs membrane
250 nm thick, 800 μm long, 30 μm wide
1 row of holes missing

Lattice spacing
a=410 nm

Pink areas: a=400 nm

Song, Noda, Asano, Akahane, Nature Materials
Why a cavity?
Simulated mode

Mode intensity
Cycle averaged $|E|^2$

Mode volume $1.2 (\lambda/n_{GaAs})^3$

Exceptionally small cavity
Very high $Q$, up to $10^6$
Narrow cavity resonance

Tip: pulled glass fiber ~ 100 nm

Laser: grating tunable diode laser
20 MHz linewidth ($10^{-7} \lambda$) around 1565 nm
Refraction

Generic solution steps:
1) Plane waves in each medium
2) Use $k_\parallel$ conservation to find allowed waves
3) Use causality to keep only outgoing waves $\rightarrow$ refracted $k$
4) Match field continuity at boundary to find $r$ and $t$
Folding bands of 1D system

Bloch wave with wave vector $k$ is equal to Bloch wave with wave vector $k + m2\pi/a$
In 2D free space, the dispersion $\omega = c|k|$ looks like a circle at any $\omega$.

Periodic system: repeated zone-scheme brings in new bands.

At crossings: coupling splits bands.
Observation of band folding

Angle resolved map of fluorescence emitted into modes of the 2D plane

Common application: LED light extraction
Measurement of gap

Folded band structure of a surface plasmon crystal
Probed dispersion at a single $\lambda$
A single incident beam can split into multiple refracted beams.

Group velocity = direction of energy flow *not* along $\mathbf{k}$. 
Refraction

Superprism: exceptional sensitivity to incidence $\theta$ and $\lambda$
Supercollimation: exceptional non-sensitivity to incidence $\theta$ and $\lambda$
A tightly focused beam has many $\Delta k$, and should diffract

Supercollimation: beam stays collimated because $v_g$ is flat

*Note: there is no guiding defect here*
Superprisms

Application: a minute change in $\omega$ is a huge change in $\theta$

‘Wavelength demultiplexer’ – note the negative refraction
Conclusions

Diffraction
• Photonic crystals diffract light, just like X-ray diffraction
• Unlike X-ray diffraction, the bandwidth is ~ 20%, not $10^{-4}$

Propagation
• Light has a nontrivial band structure – similar to e- band-structure
• Dispersion surfaces are like Fermi surfaces
• Light has polarization. Photons do not interact with photons
• Band structure controls refraction and propagation speed
• Band gap: light does not enter. No states in the crystal

Defects
• line defects guide light
• Point defects confine light for up to $10^6$ optical cycles in a $\lambda^3$ volume