Nanophotonics 2016
Problem set 3

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Hand in: April 26^{th}, 2016
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1 Ray tracing of a perfect lens

First we consider a normal lens, governed by the lens formula:

$$\frac{1}{v} + \frac{1}{b} = \frac{1}{f}$$

that relates the distance v of object to lens to the distance b of image to lens via the focal length f of the lens. Suppose we make an image of an object in an object plane at distance v of the lens using a positive lens of focal distance f.

(a) What is the orientation of the image in the image plane? What happens if the object distance is less than f?

(b) Make a plot of the magnification of the object versus object distance v/f (in units of f), and of the total distance $b + v$ between object and image versus v.

Now we consider a slab of material with refractive index $n = -1$ and thickness d. Using ray tracing based on Snells law.

(c) What is the magnification and orientation of the image if the object is at a distance $d/2$ of the slab surface? Show a diagram that traces the rays from object to image. Notice the virtual image inside the slab.

(d) What happens with the magnification and object-to-image distance when the object is brought closer to the lens? Is an image still formed if the object is moved further away from the lens?

2 Magnetic-atom design

Consider the magnetic atom design in figure 1 (a split ring). Using 1^{st} year electrodynamics (see University Physics book) and the illustration given in figure 1b, assuming that $l = 1$, $w = 0.25l$, $t = 0.2l$, $d = 0.3l$, $\epsilon_c = 1$,

(a) Calculate the capacitance C and the inductance L. How does the resonance scale with size l? If you want to have a resonance at 1.5 micron wavelength, how small should the split ring be?

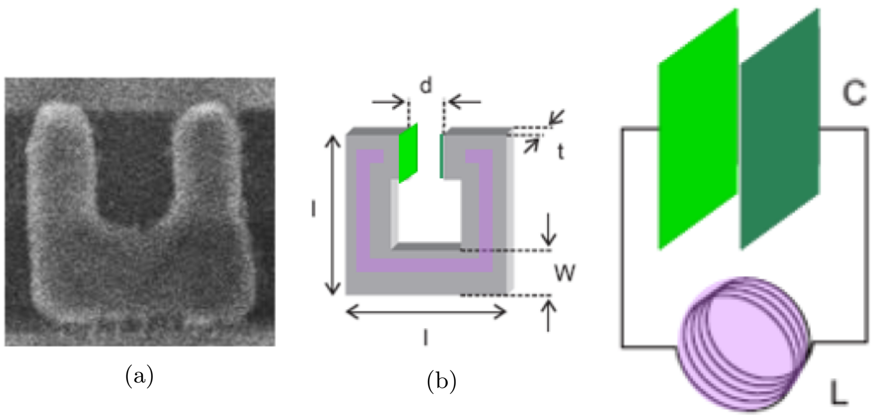
(b) Draw an equivalent circuit including Ohmic resistance R. Show that the circuit impedance has a Lorentzian shape. Derive the resonance and quality factor.

(c) Estimate the resistance through $R = \frac{\rho \cdot l_{\text{eff}}}{A}$. Remember that $\rho = \frac{1}{\sigma}$, and $\epsilon = 1 + \frac{i\sigma}{\omega}$. What is the expected quality factor at 1.5 micron for a gold SRR?

(d) We want to estimate the effective magnetic permeability μ_{eff} of a lattice of SRRs. We start by finding the circulating current I in the SRR. An external magnetic field $\vec{H} = H_0 e^{-i\omega t}$ drives a current I in the ring, which is related through Ohms law $V = I \cdot Z$ to the electromotive potential V. The electromotive V results from Faradays law of induction $V = -\oint \frac{d\vec{H}}{dt}\mu_0 dA = i\omega\vec{H}\mu_0 l^2$. The induced current gives rise to individual magnetic dipole moments $\vec{m} = I \cdot l^2 = \alpha \cdot \vec{H}$ that contribute to the microscopic magnetic field $B_{\text{eff}} = \mu_0(\vec{H}+\vec{M}) = \mu_0\left(\vec{H} + \frac{N}{V}\vec{m}\right) = \mu_0\mu_{\text{eff}}\vec{H}$, where $\frac{N}{V}$ is the magnetic dipole moment number density with N individual magnetic dipole moments per volume V.

  (i) Find the magnetic dipole moment $\vec{m}$ and the magnetic polarizability α. How large do you think the polarizability is compared to the split ring volume for the split ring in figure 1?

  (ii) Obtain the effective magnetic permeability μ. Fill in numbers of your choice for SRRs with $l = 200nm$ with a cubic lattice constant $a = 300nm$ and plot the magnetic permeability versus size.



(a) (b) (c)

Figure 1: a) shows a scanning electron microscope (SEM) image of a split ring resonator (SRR) made of Au on a SiO_2 substrate by electron beam lithography. The approximate dimensions of the SRR, as depicted in b), are $l = 200nm$, $w = 80nm$, $t = 30nm$, and $d = 80nm$, where l=length, w=width, t=thickness and d=gap size of the SRR. As depicted in c), an SRR can be described as an LC-circuit with a parallel plate capacitance C and inductance L of a single-loop coil. These parameters depend on the SRR geometry.